# Babar anomaly and the pion form factors

Adam Szczepaniak
Indiana University

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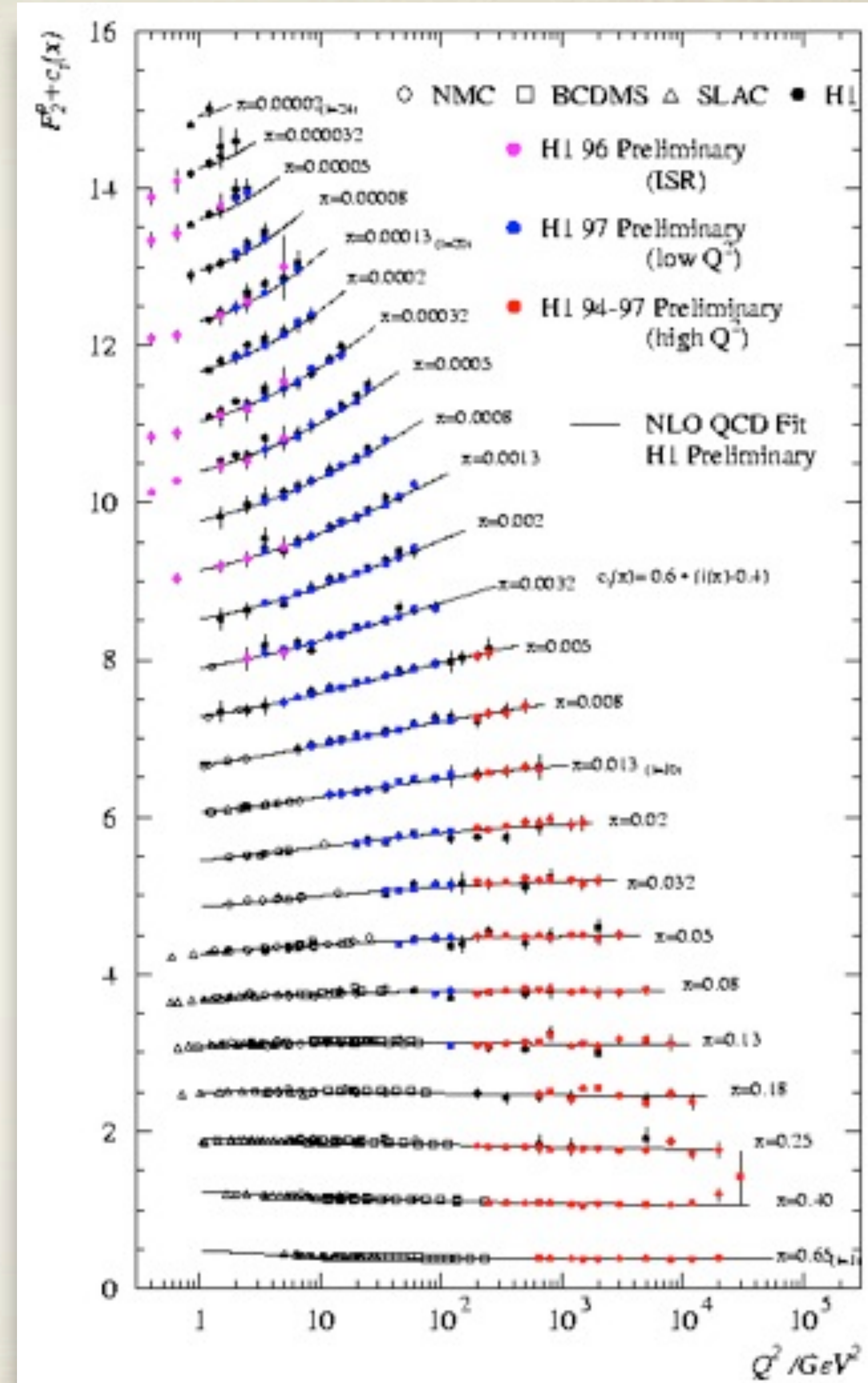

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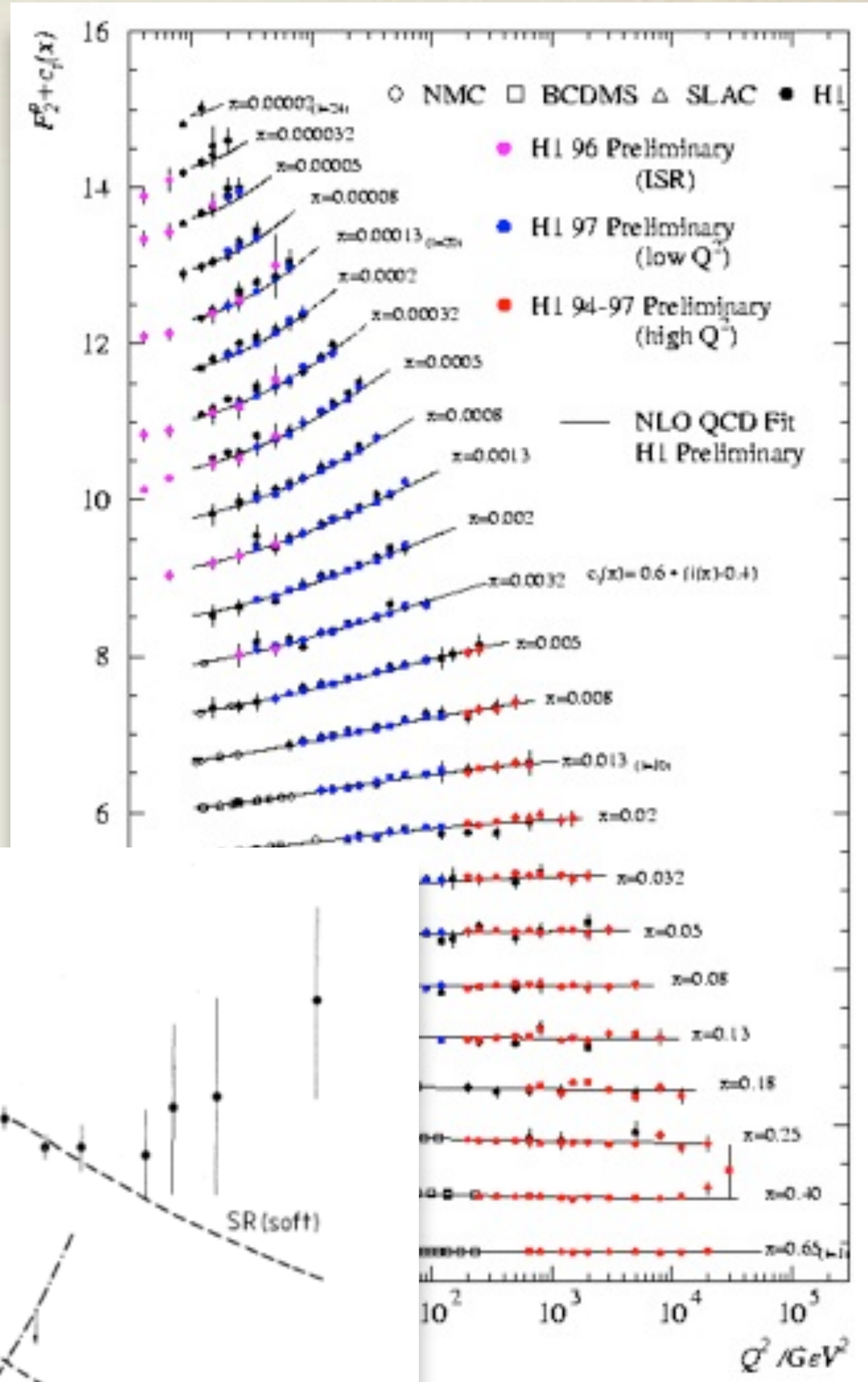
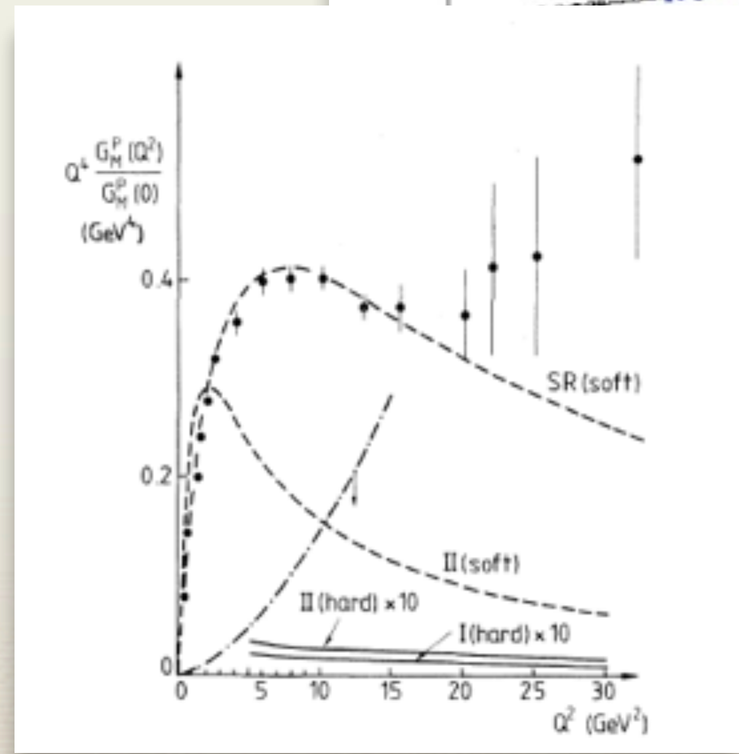



- Form-factors :: quark/gluon structure of hadrons

- high $Q^2$ :: LO pQCD (twist/$\alpha_s$ expansion)

- Questions:

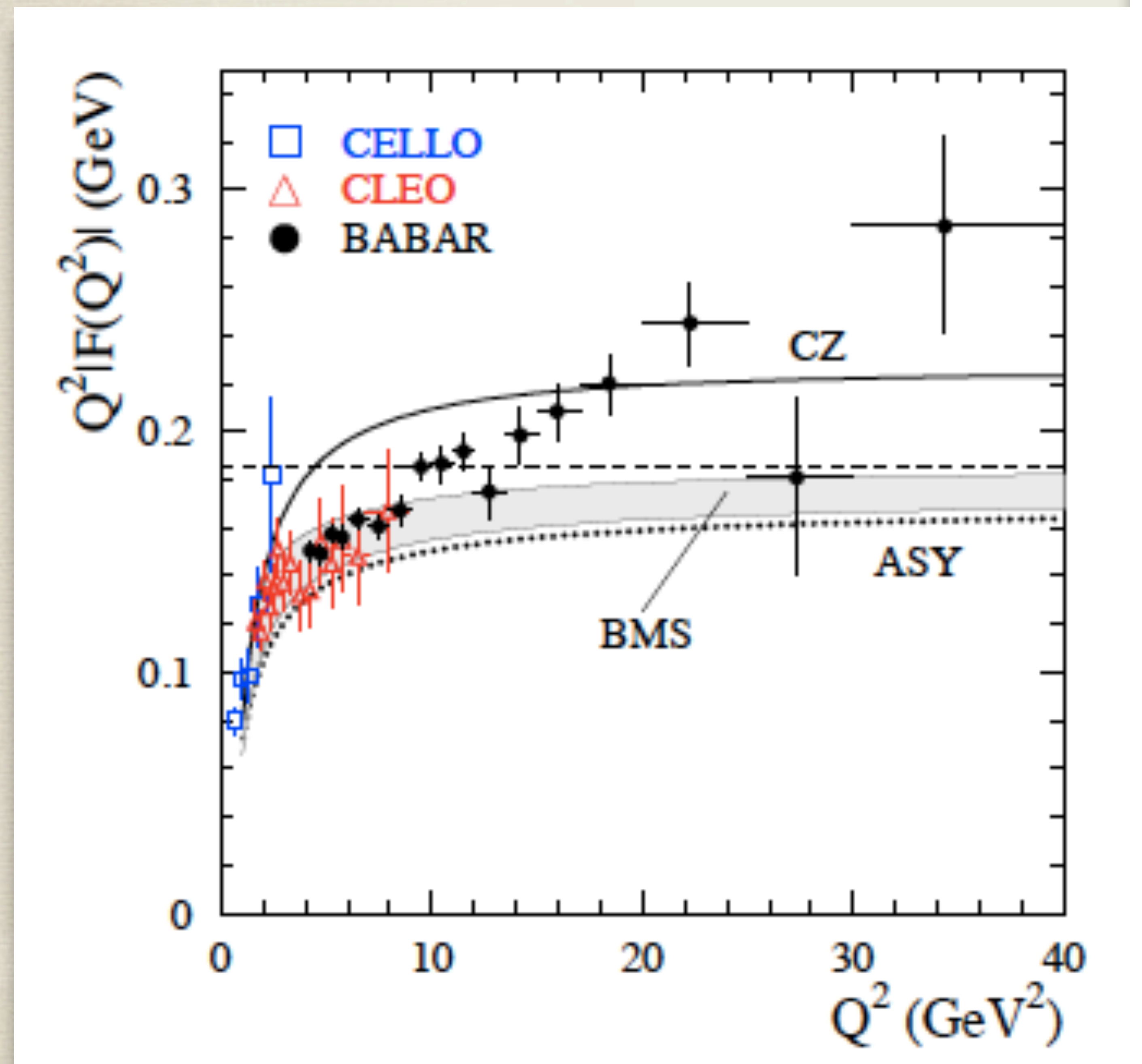
  1. Role of hadronic vs partonic d.o.f
  2. Is there indication that all-orders re-summation is needed

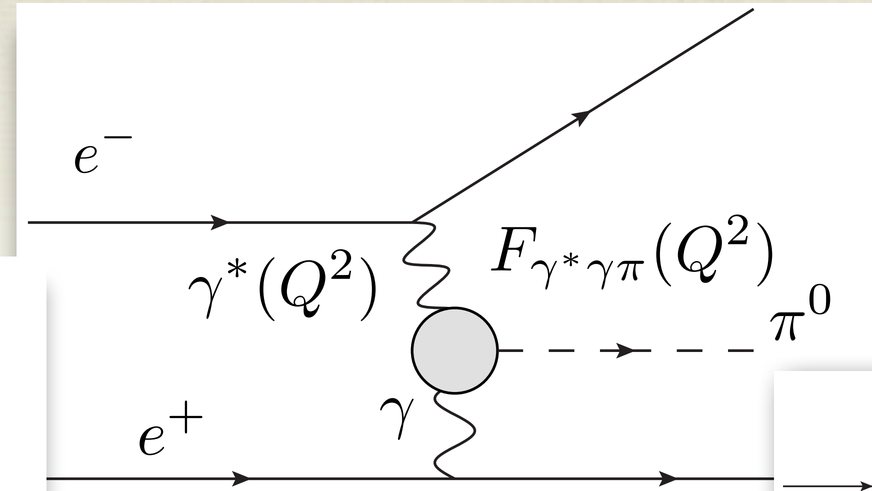


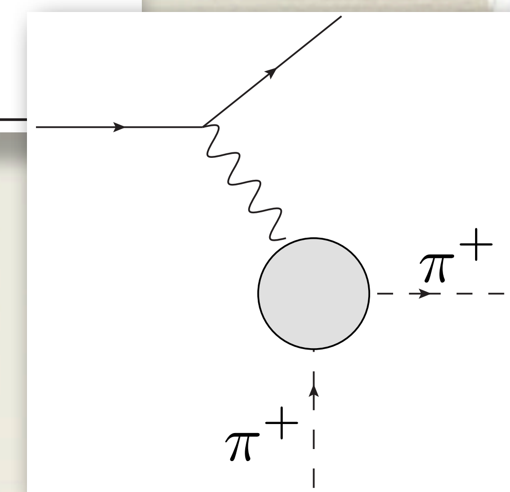

with M.Gorshtein, P.Guo, J.L.Londergan, F.L.Estrada

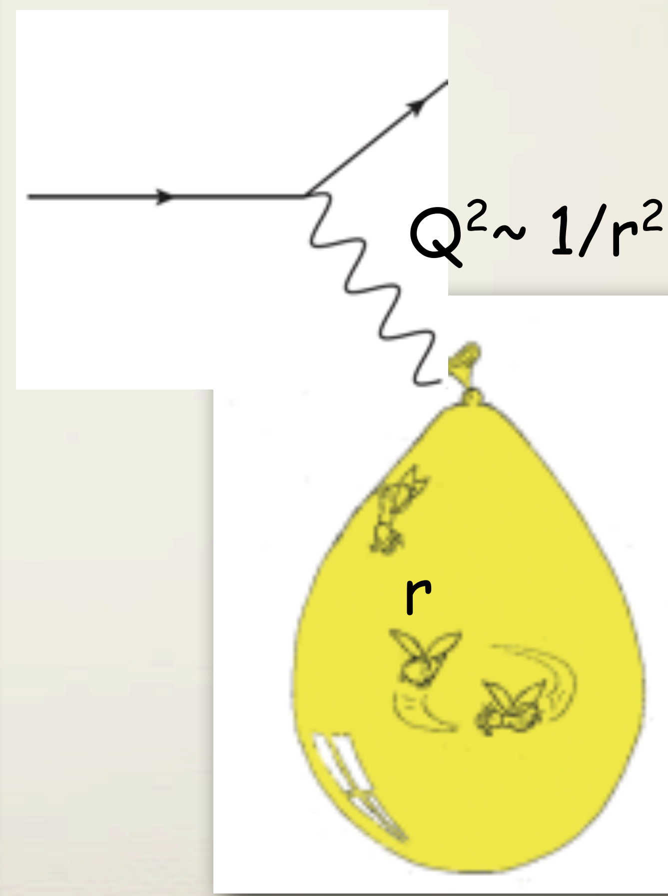
BaBar anomaly $e^+e^- \to \pi^0 e^+ e^-$

B.Aubert et al. Phys.Rev. (2009)




theory: G.P.Lepage, S.Brodsky

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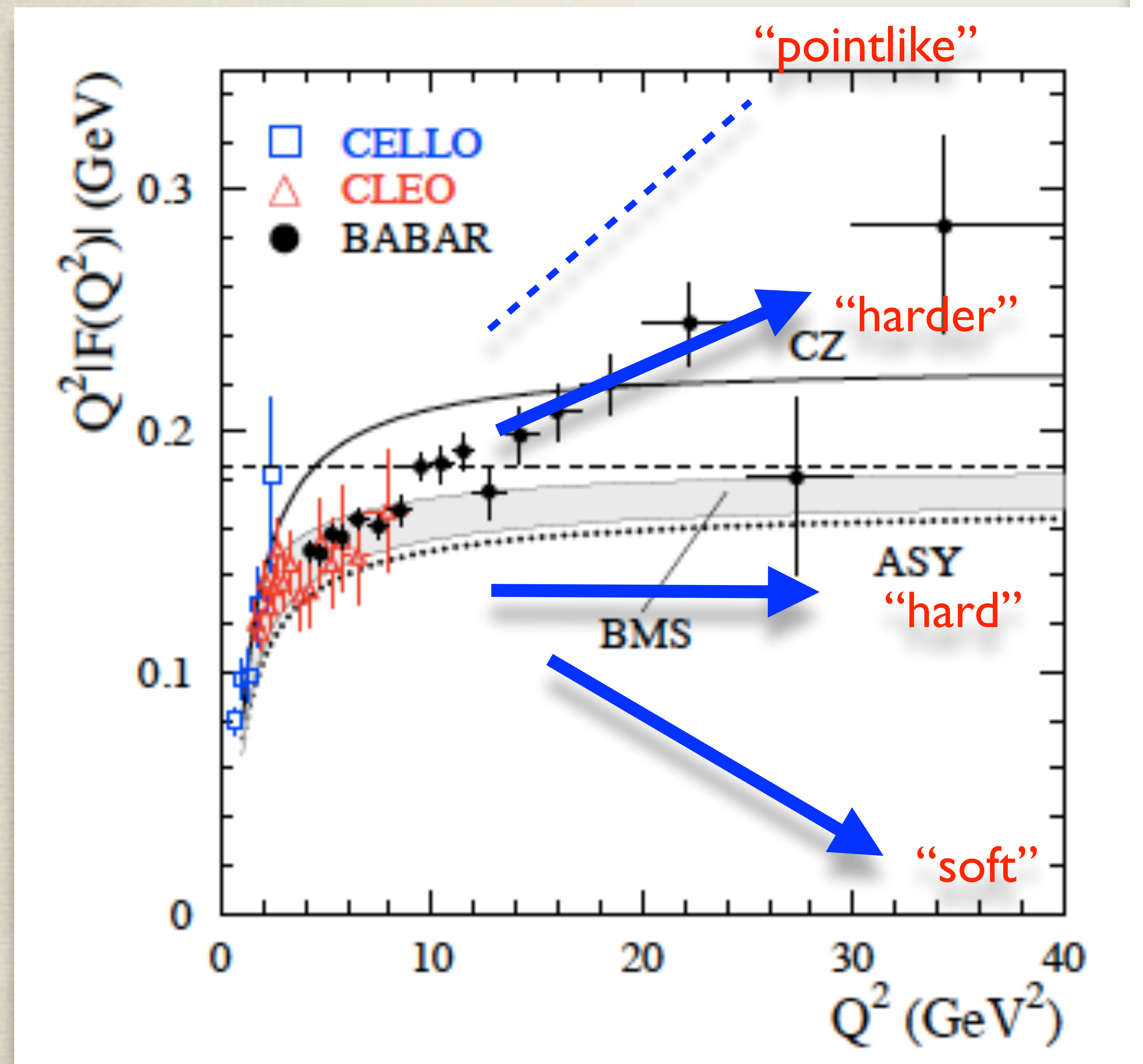
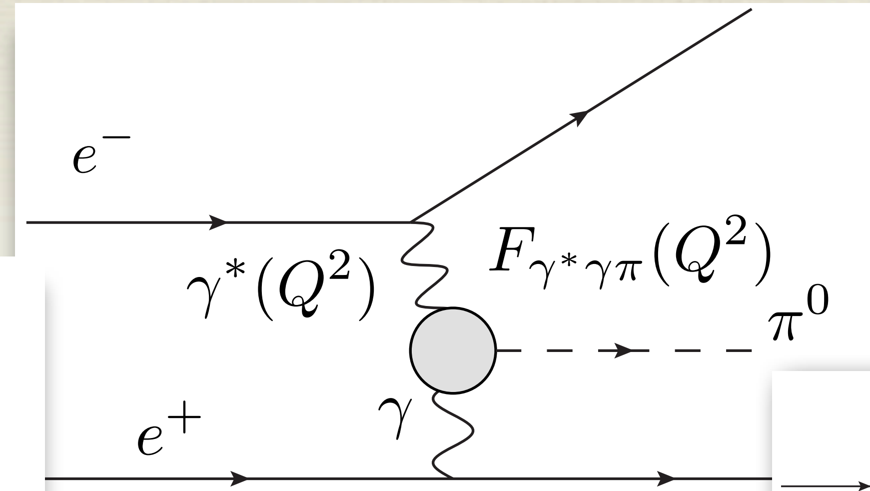
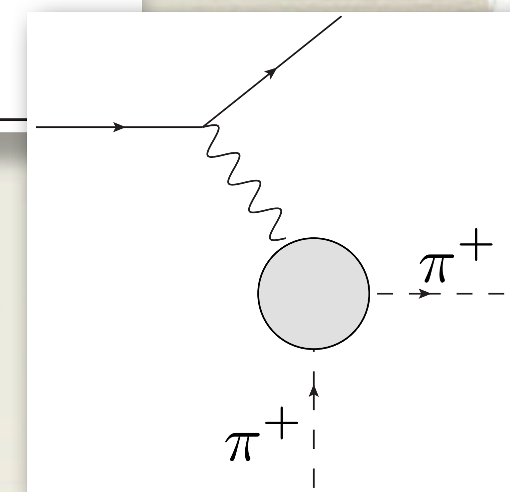
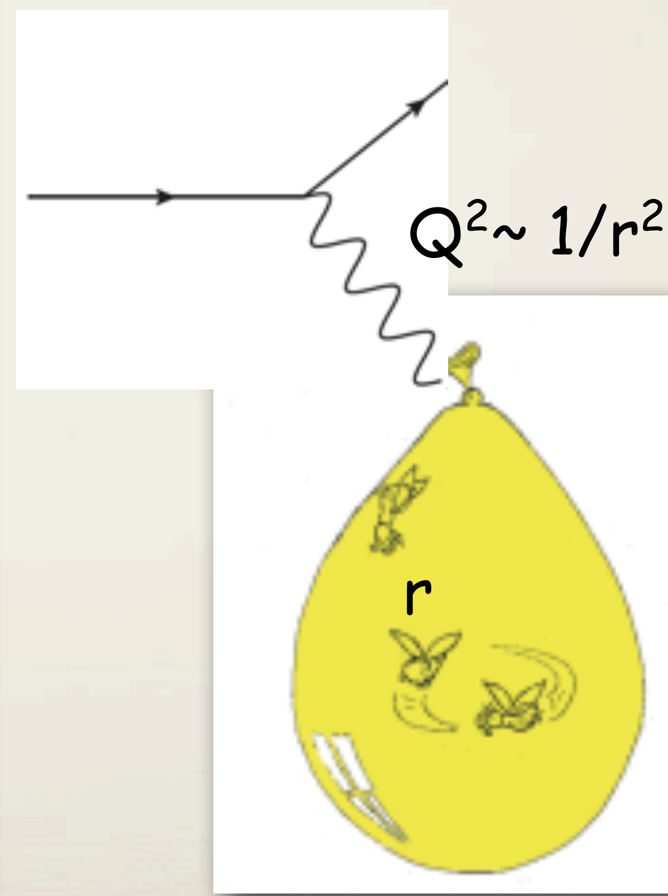


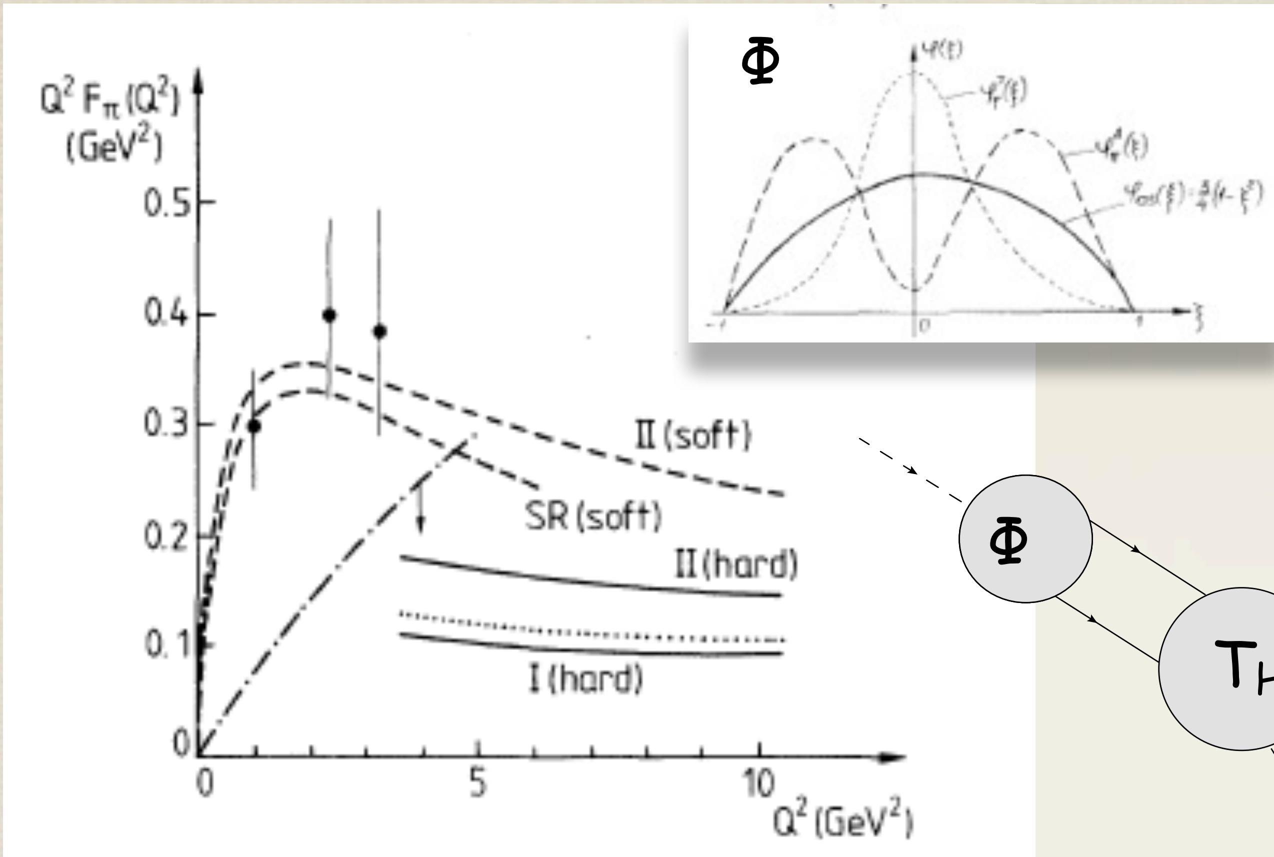



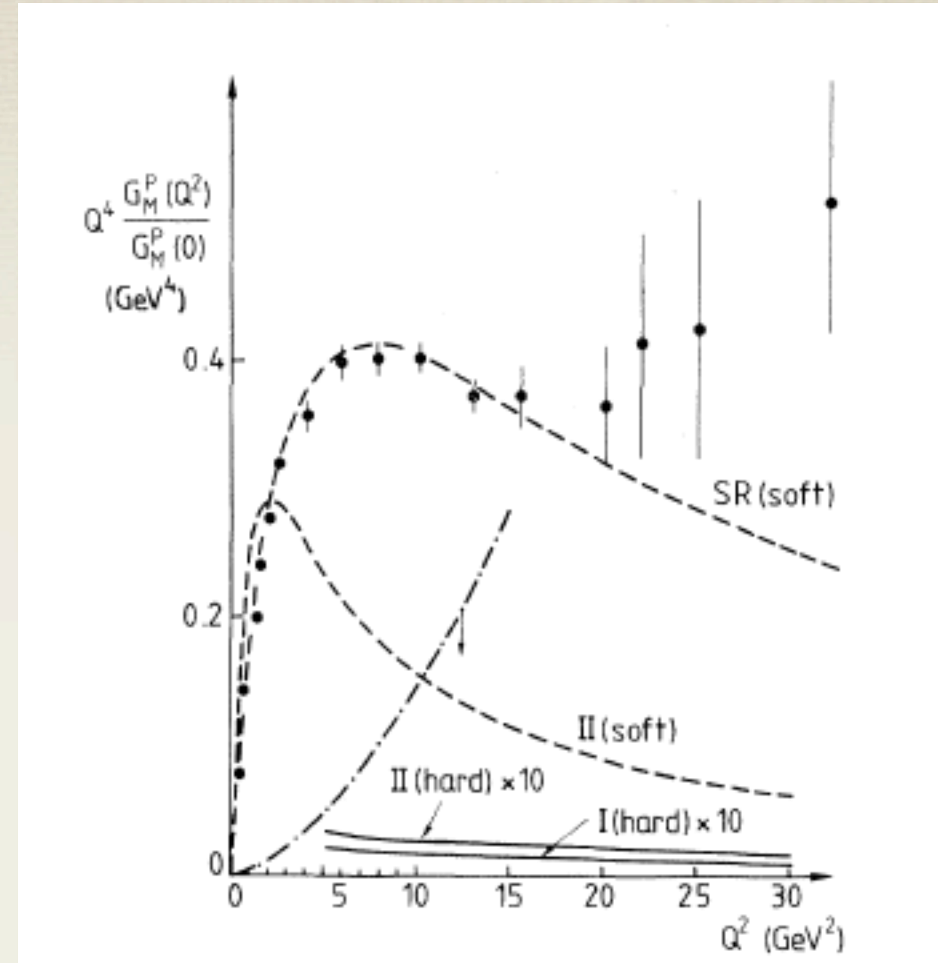



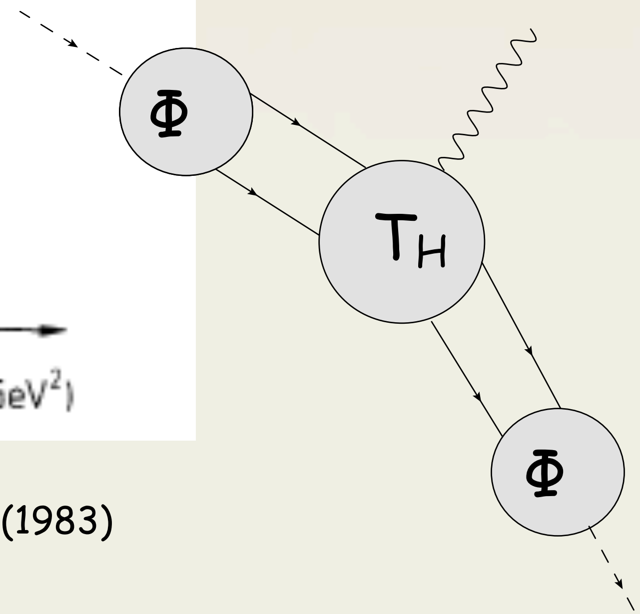




theory: G.P.Lepage, S.Brodsky

# problems with LO pQCD in exclusive reactions



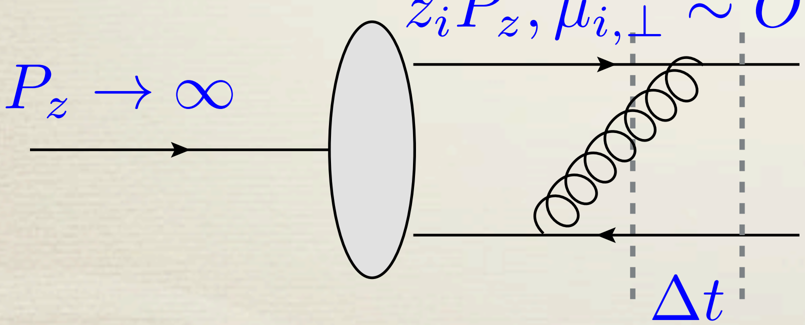




N.Isgur, C.H.Llewellyn Smith, Phys.Rev.Lett.(1983)

$$\frac{1}{\Delta t} \sim \Delta E = \sum_i \frac{\mu_{i\perp}^2}{z_i P_z}$$

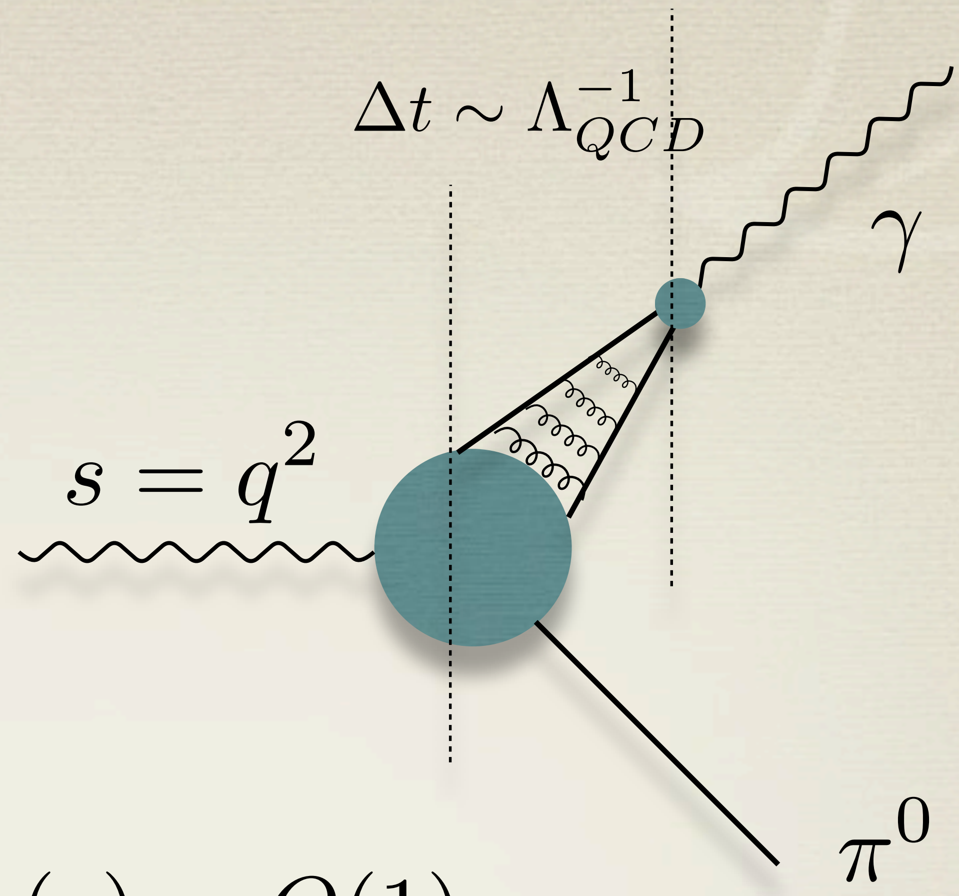
$z_i P_z, \mu_{i,\perp} \sim O(1)$

$P_z \to \infty$

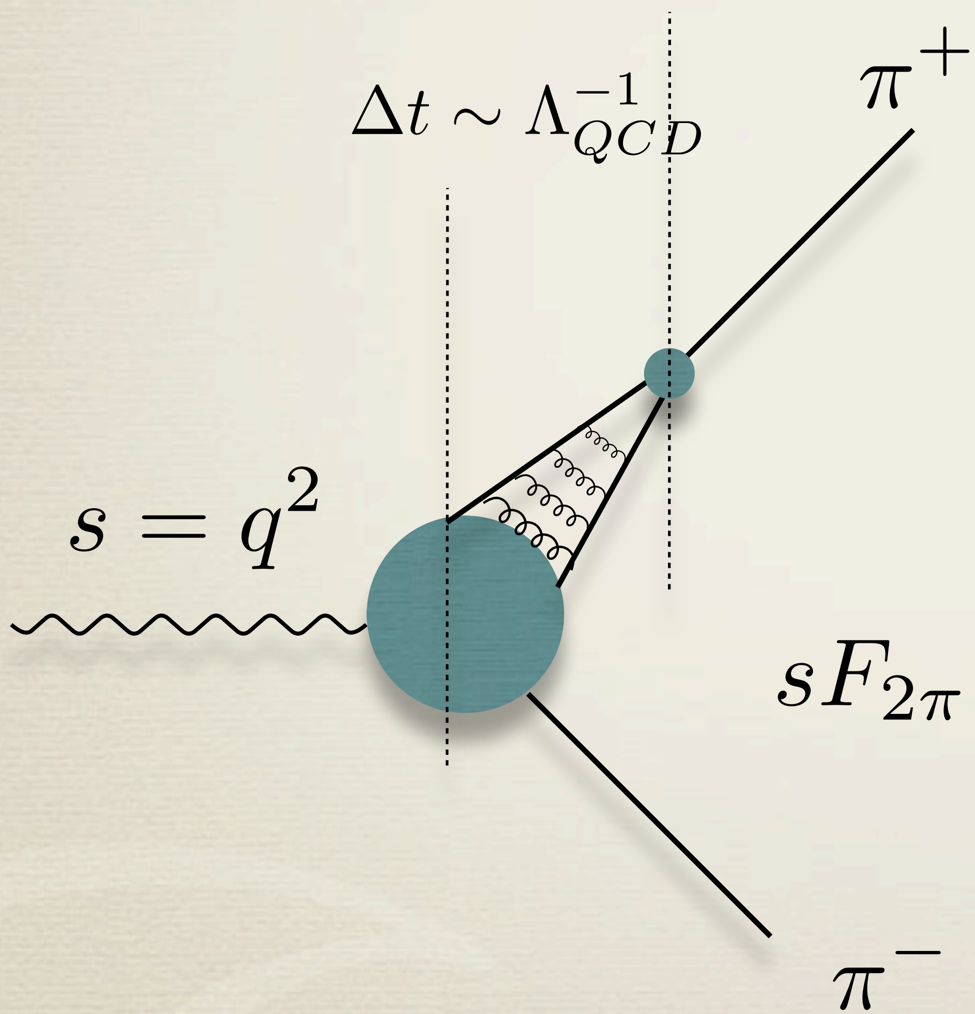



valid for $z_i$ $P_z$ large i.e. NOT in the end-point region

similar final states but
different asymptotic
predictions




$$sF_{\gamma\pi}(s) \sim O(1)$$

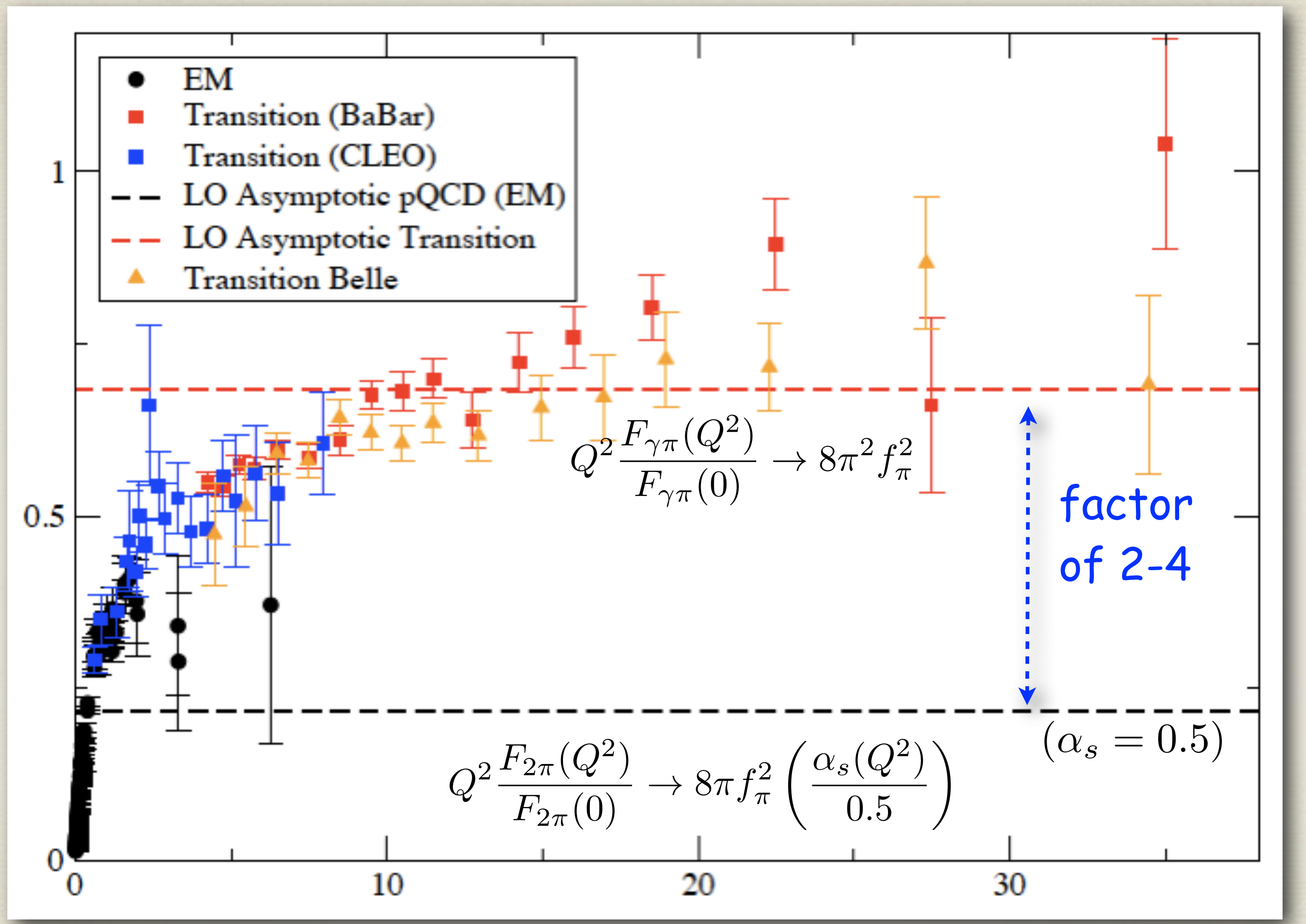


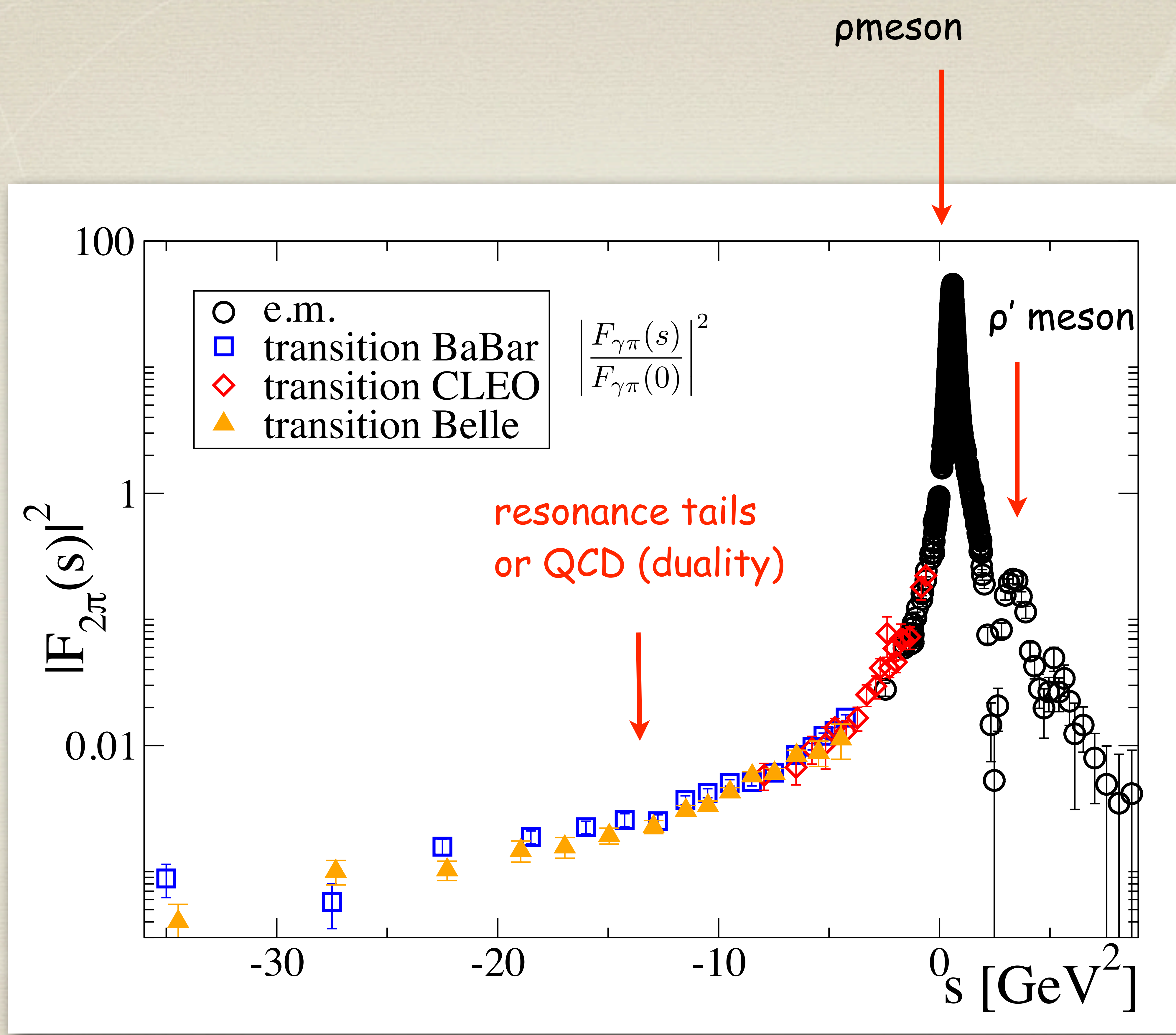

$$sF_{2\pi}(s) \sim O(\alpha_s(Q^2))$$

... but it does look different on
the Light Front

S.Brodsky,P.Lepage
A.Radyushkin, A.Efremov

## * Pion form factors : still a mystery



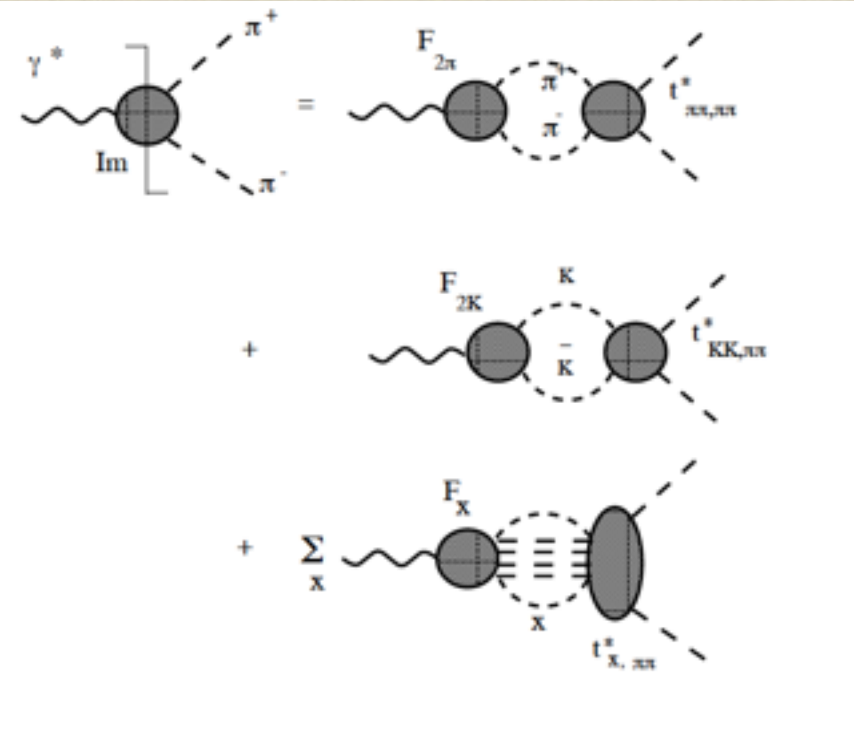

ρmeson
ρ' meson
resonance tails
or QCD (duality)
e.m.
transition BaBar
transition CLEO
transition Belle
$\left|\frac{F_{\gamma\pi}(s)}{F_{\gamma\pi}(0)}\right|^2$
$|F_{2\pi}(s)|^2$
100
1
0.01
-30
-20
-10
0
s [GeV$^2$]

* Dispersive analysis

$$F(s) = F(0) + \frac{s}{\pi} \int_{s_{th}} ds' \frac{\text{Im } F(s')}{s'(s'-s)}$$

$$F = F_{2\pi}, F_{\gamma\pi}$$

# * Dispersive analysis




$$F(s) = F(0) + \frac{s}{\pi}\int_{s_{th}} ds' \frac{\text{Im } F(s')}{s'(s'-s)}$$

$$F = F_{2\pi}, F_{\gamma\pi}$$

EM F.Factor

$$\text{Im } F_{2\pi}(s) = t^*_{2\pi,2\pi}\rho_{2\pi}F_{2\pi} + t^*_{2\pi,K\bar{K}}\rho_{2K}F_{2K} + \sum_X t^*_{2\pi,X}\rho_X F_X$$

$$= t^*\rho F_{2\pi} + R$$

$$t(s) = \int dz_s t^{I=1}(s,t(z_s))P_1(z_s) \qquad \rho(s) = (1 - s/s_{th})^{1/2}$$

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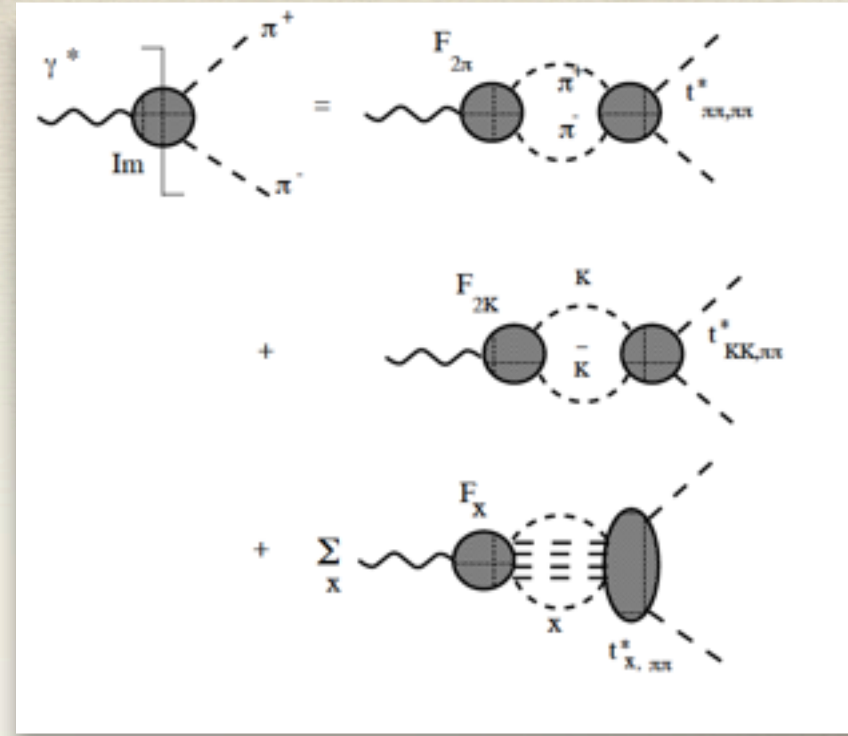


$$F = F_{2\pi}, F_{\gamma\pi}$$

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$$F_{2\pi}(s) = 1 + \frac{1}{\pi}\int_{s_{th}} ds' \frac{t^*(s')\rho(s')F_{2\pi}(s') + R(s')}{s'(s'-s)}$$

(so far) Exact representation of the electromagnetic form factor

## * Dispersive analysis cont.

elastic — inelastic $R(s') \propto \theta(s' - (4m_\pi)^2)$

$$F_{2\pi}(s) = 1 + \frac{1}{\pi}\int_{s_{th}} ds' \frac{t^*(s')\rho(s')F_{2\pi}(s') + R(s')}{s'(s'-s)}$$

solution

# * Dispersive analysis cont.

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$$F_{2\pi}(s) = 1 + \frac{1}{\pi}\int_{s_{th}} ds' \frac{t^*(s')\rho(s')F_{2\pi}(s') + R(s')}{s'(s'-s)}$$

solution

$$F_{2\pi}(s) = \frac{N(s)}{D(s)}$$

$N(s)$ ← inelastic cut

$D(s)$ ← elastic cut

# * Dispersive analysis cont.

elastic  inelastic  $R(s') \propto \theta(s' - (4m_\pi)^2)$

$$F_{2\pi}(s) = 1 + \frac{1}{\pi}\int_{s_{th}} ds' \frac{t^*(s')\rho(s')F_{2\pi}(s') + R(s')}{s'(s'-s)}$$

solution

$$F_{2\pi}(s) = \frac{N(s)}{D(s)}$$

$N(s)$ ← inelastic cut

$D(s)$ ← elastic cut

$$N(s) = 1 + \frac{s}{\pi}\int_{s_i} ds' \frac{D(s')\text{Re R(s')}}{[1 - it^*(s')\rho(s')]s'(s'-s)}$$

$$D(s) = \exp\left(-\frac{s}{\pi}\int_{s_{th}} ds' \frac{\phi(s')}{s'(s'-s)}\right)$$

$$\phi = \arctan \text{Re}\ t/(1 - \text{Im}\ t\rho)$$

## * Dispersive analysis cont.

elastic

inelastic $R(s') \propto \theta(s' - (4m_\pi)^2)$

$$F_{2\pi}(s) = 1 + \frac{1}{\pi}\int_{s_{th}} ds' \frac{t^*(s')\rho(s')F_{2\pi}(s') + R(s')}{s'(s'-s)}$$

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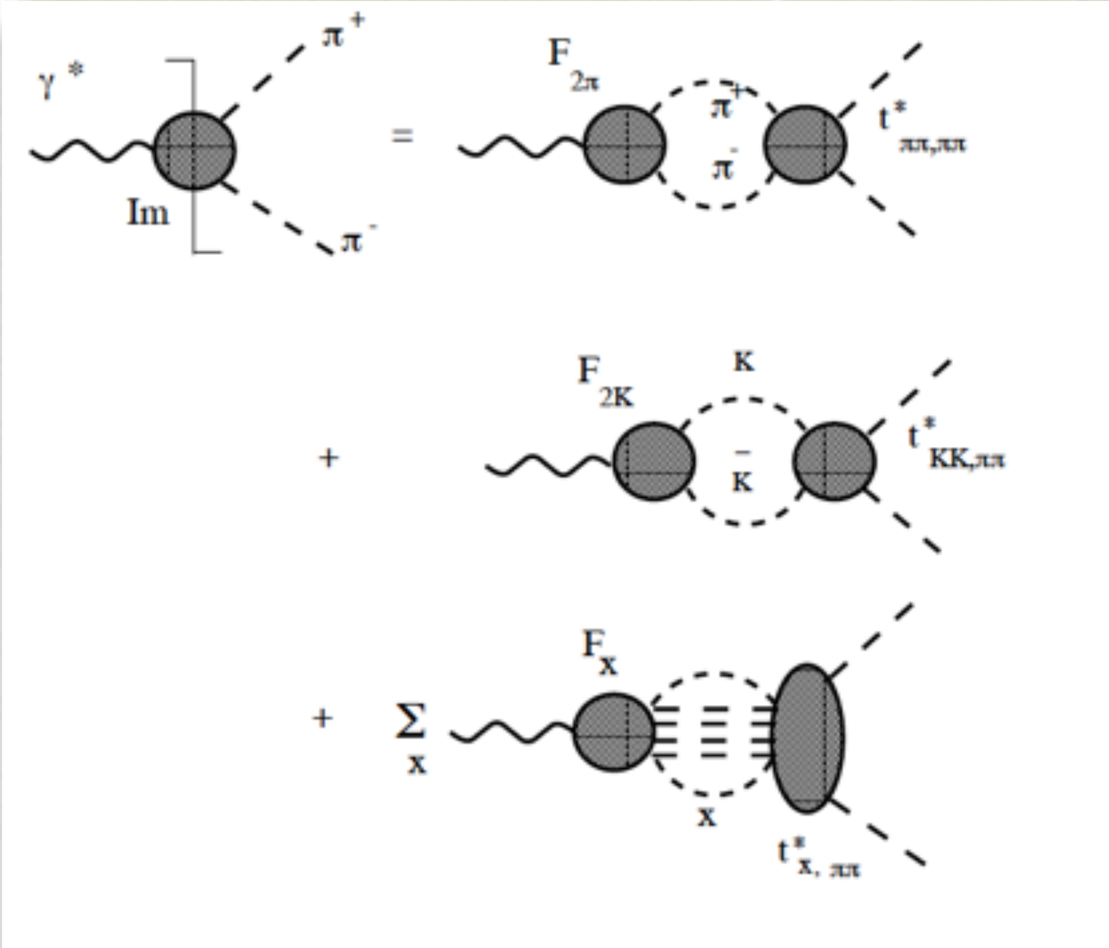
on shell P-wave ππ amplitude

$$\phi = \arctan \text{Re } t/(1 - \text{Im } t\rho)$$

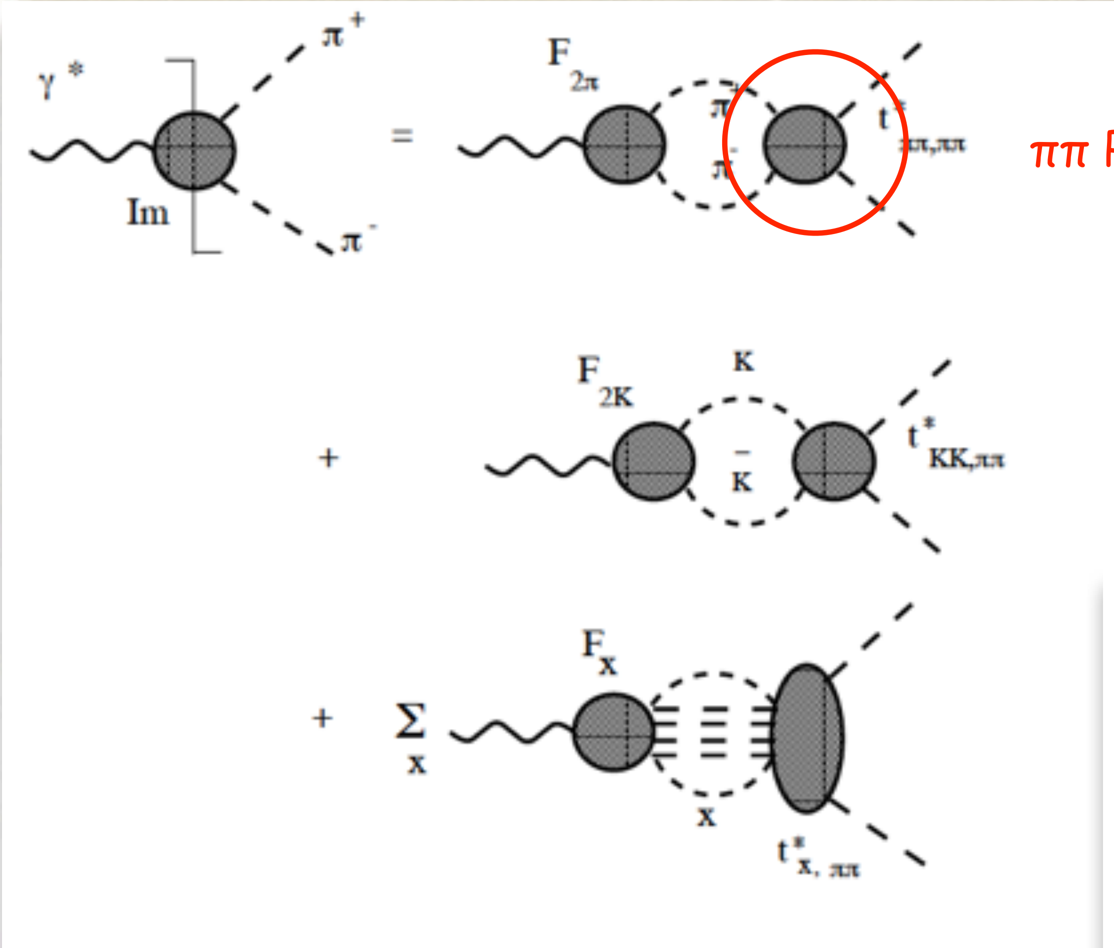
input: $t(s), R(s)$

on shell, exclusive ππ-> X amplitudes + associated form factors

output: $F_{2\pi}(s)$

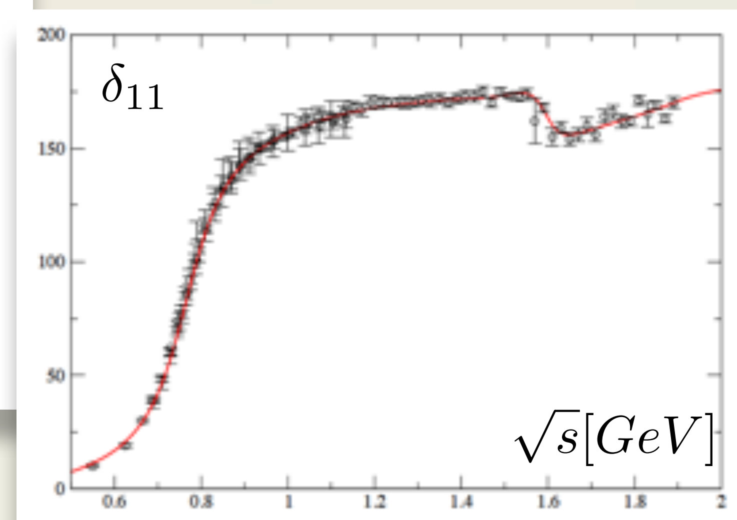


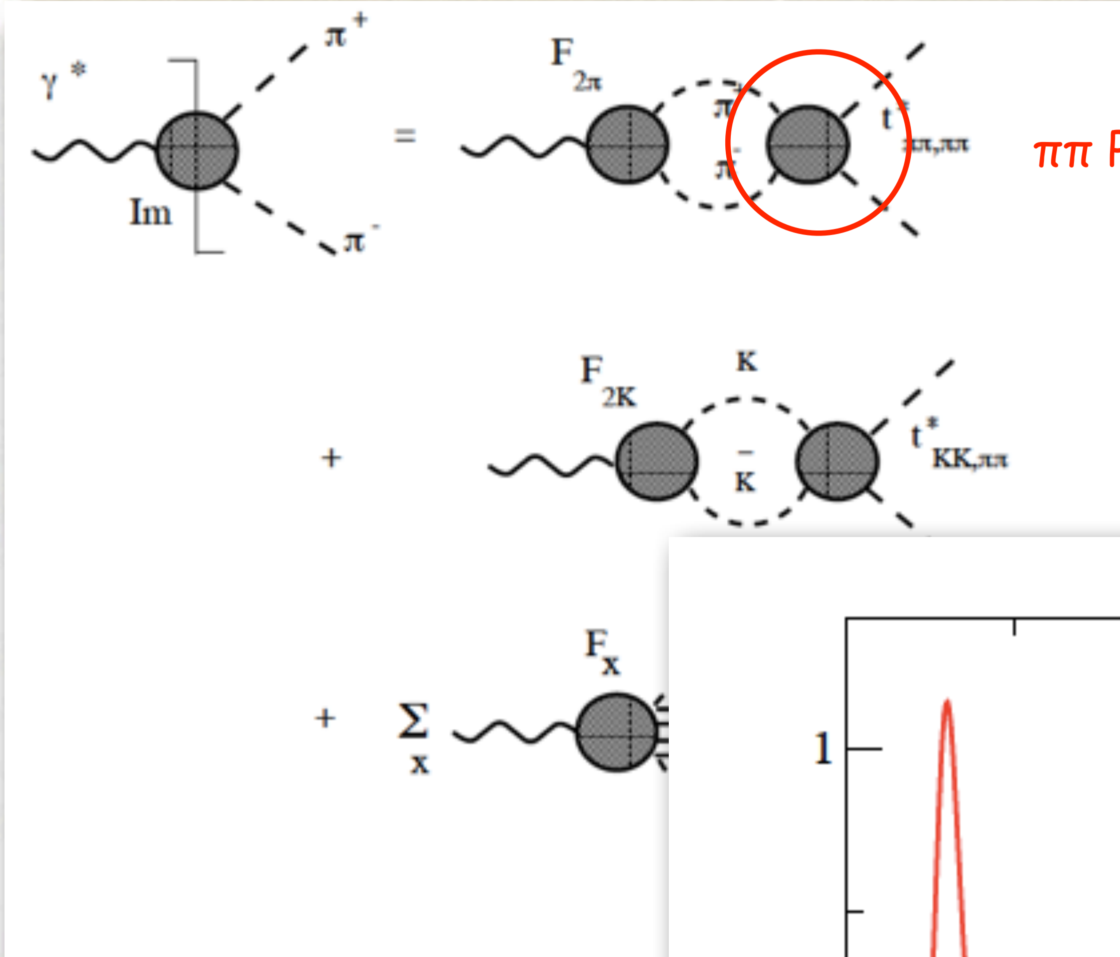
γ *
π +
π -
Im
=
F 2π
π +
π -
t* ππ,ππ
+
F 2K
K
K̄
t* KK,ππ
+
Σ x
F x
x
t* x, ππ




ππ P-wave amplitude

$$t = \frac{\eta e^{2i\delta} - 1}{2i\rho}$$



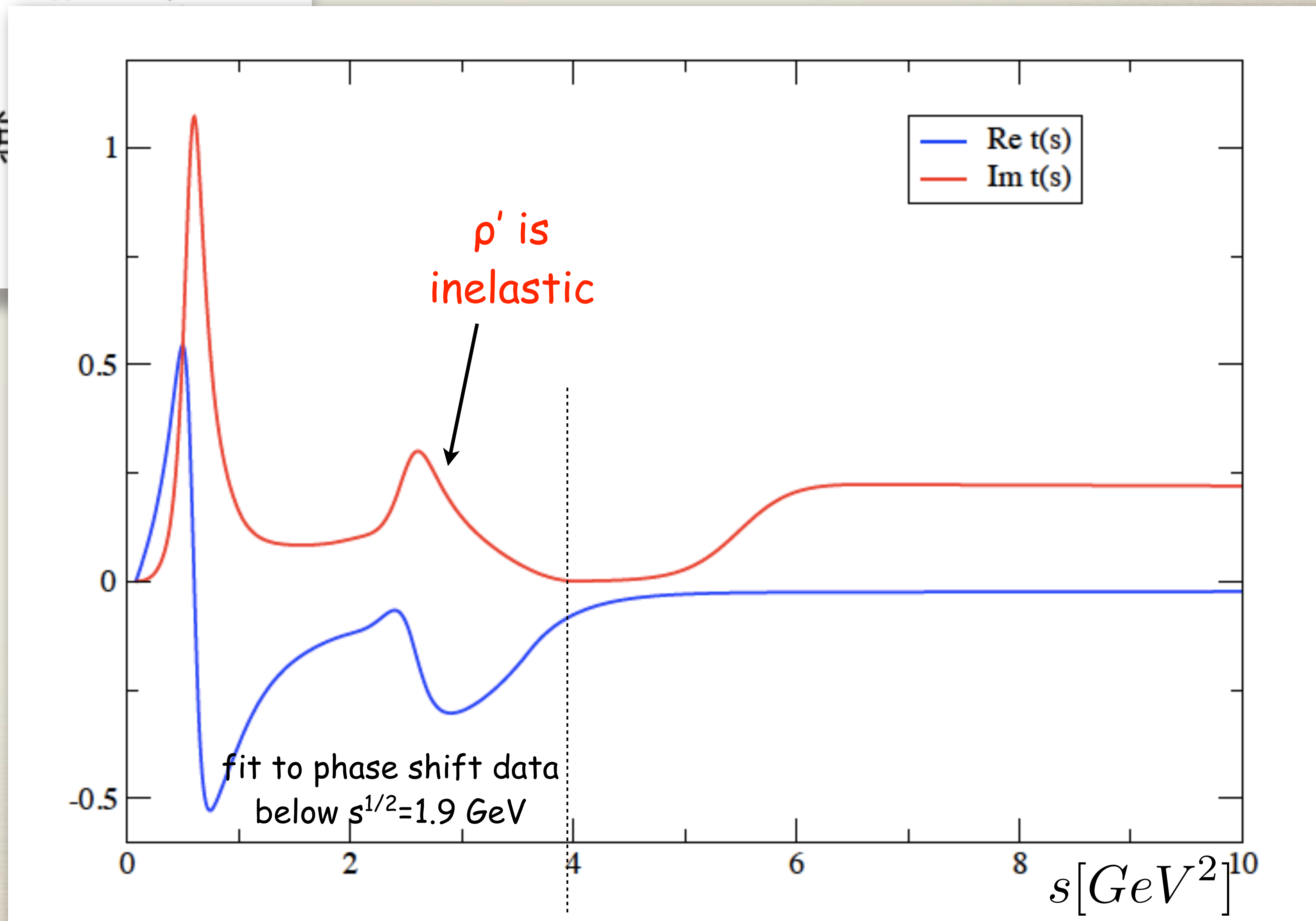


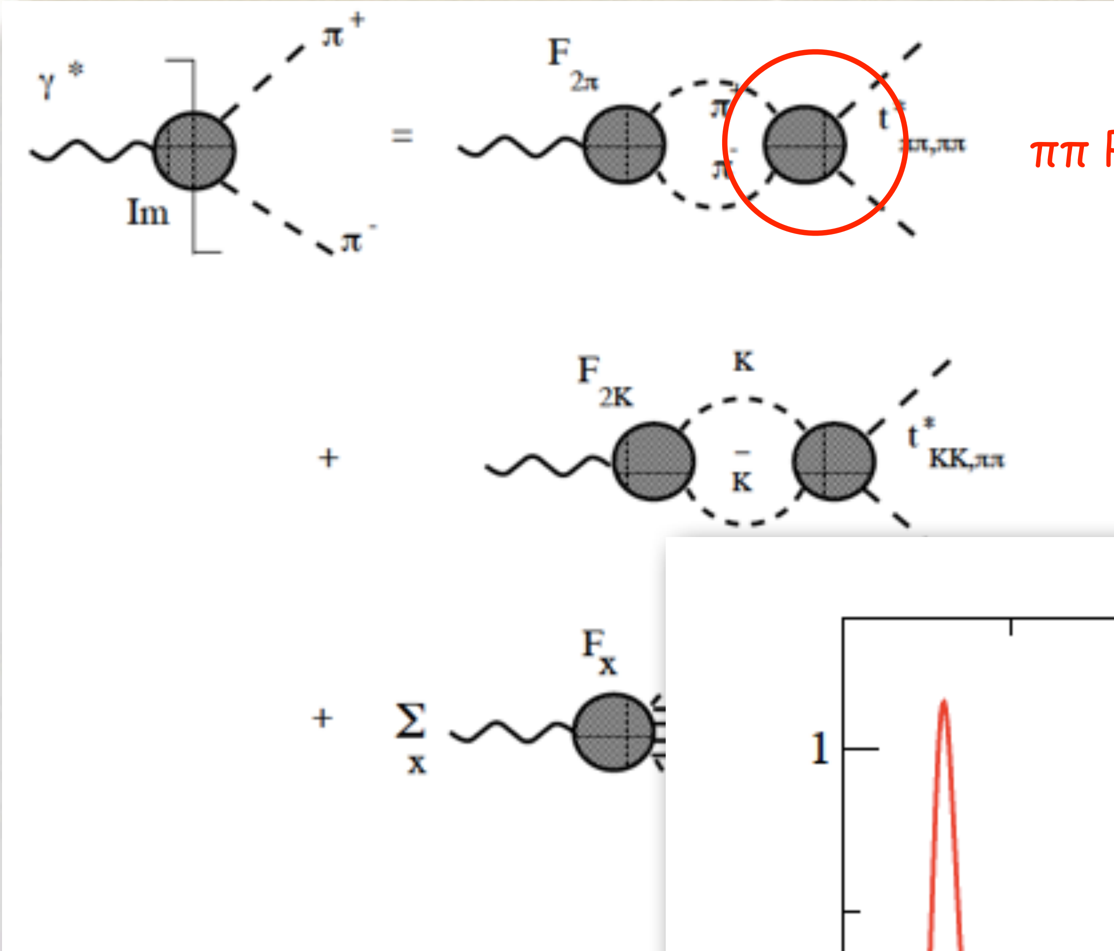

ππ P-wave amplitude

$$t = \frac{\eta e^{2i\delta} - 1}{2i\rho}$$

Im $t$

Re $t$



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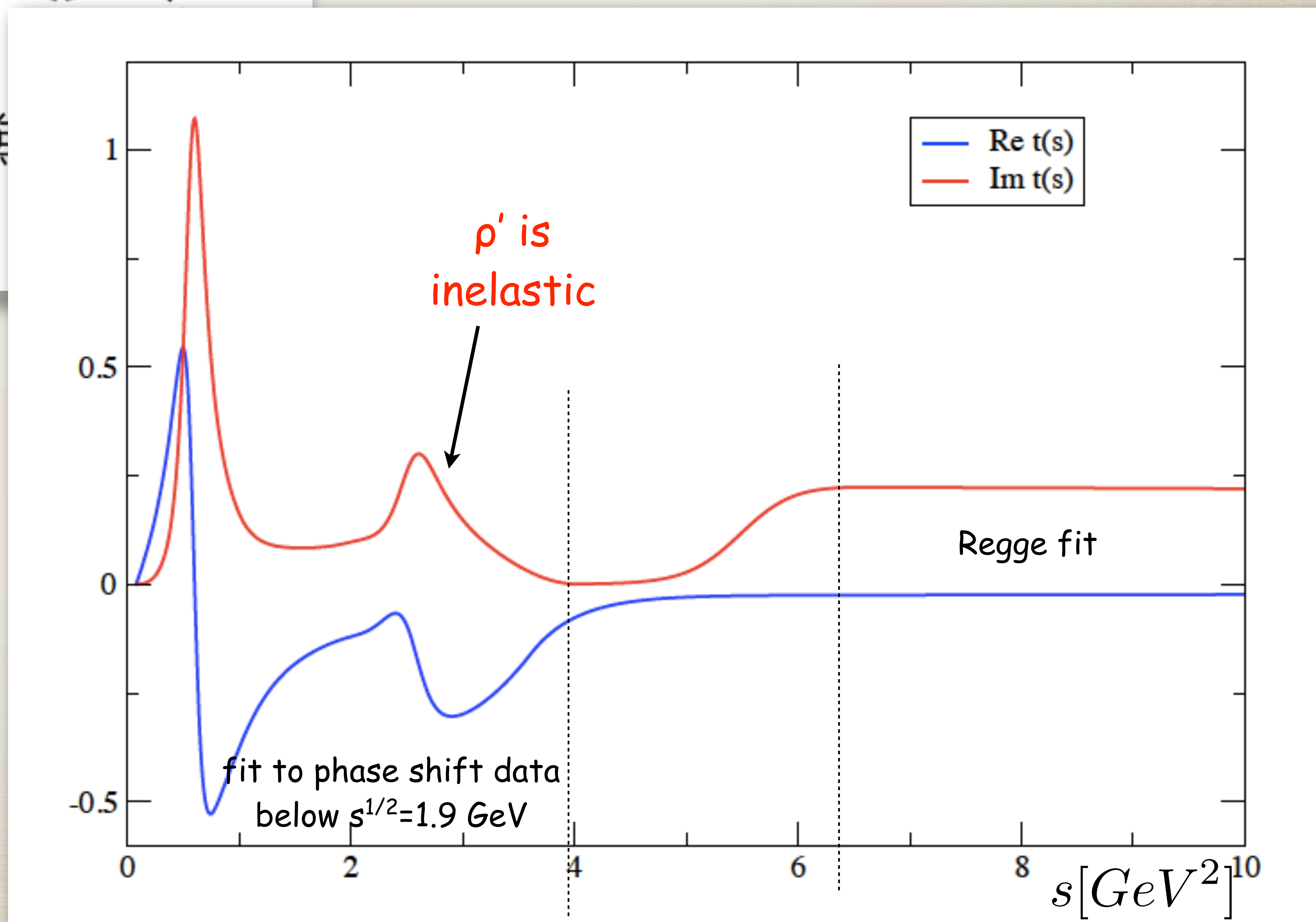

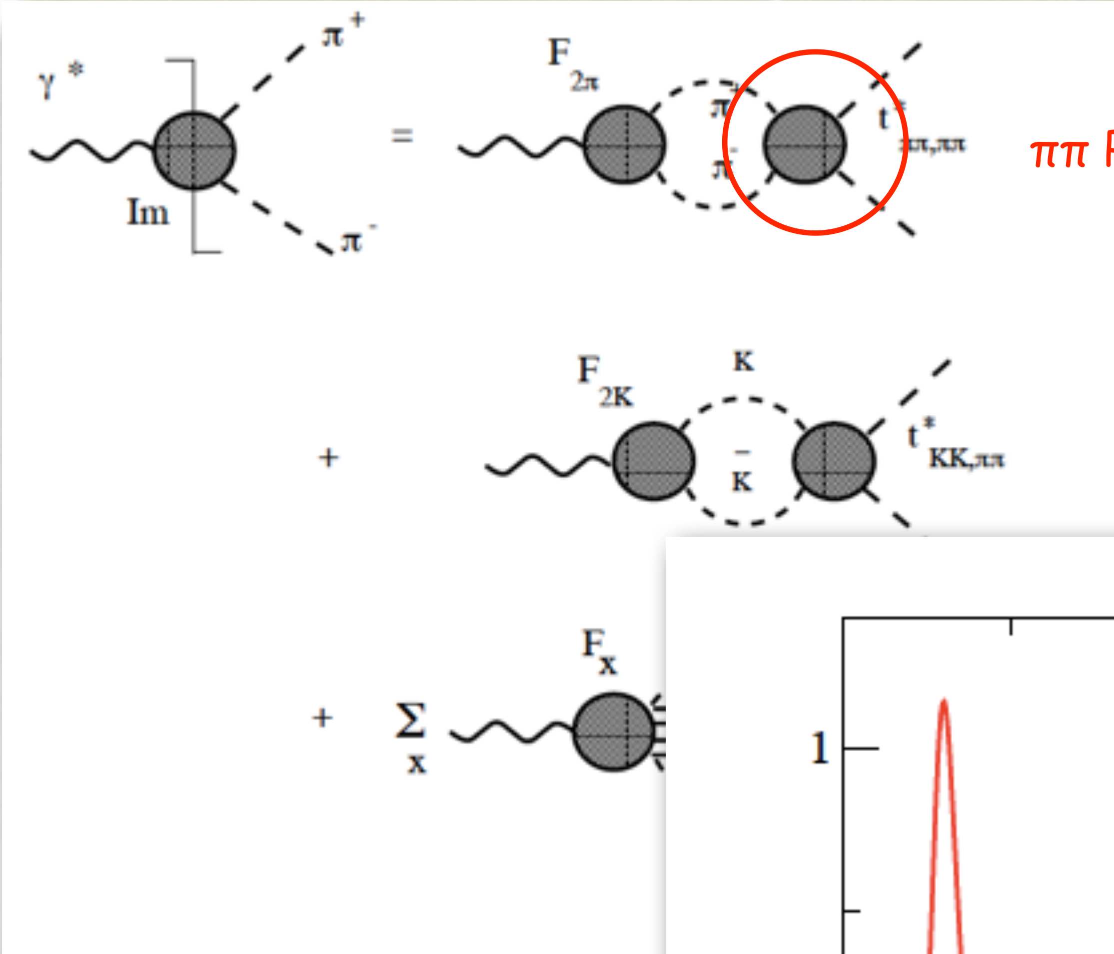


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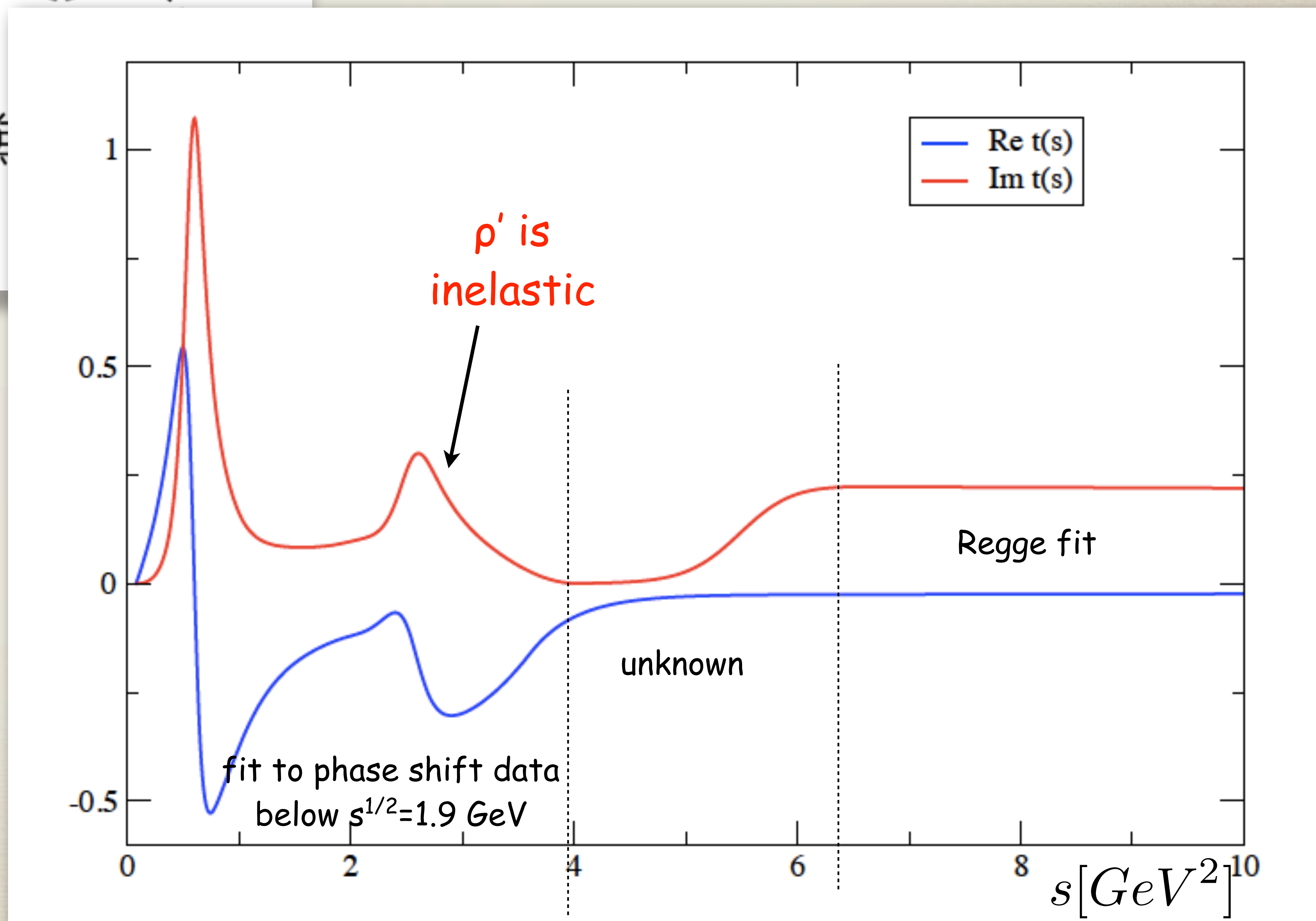
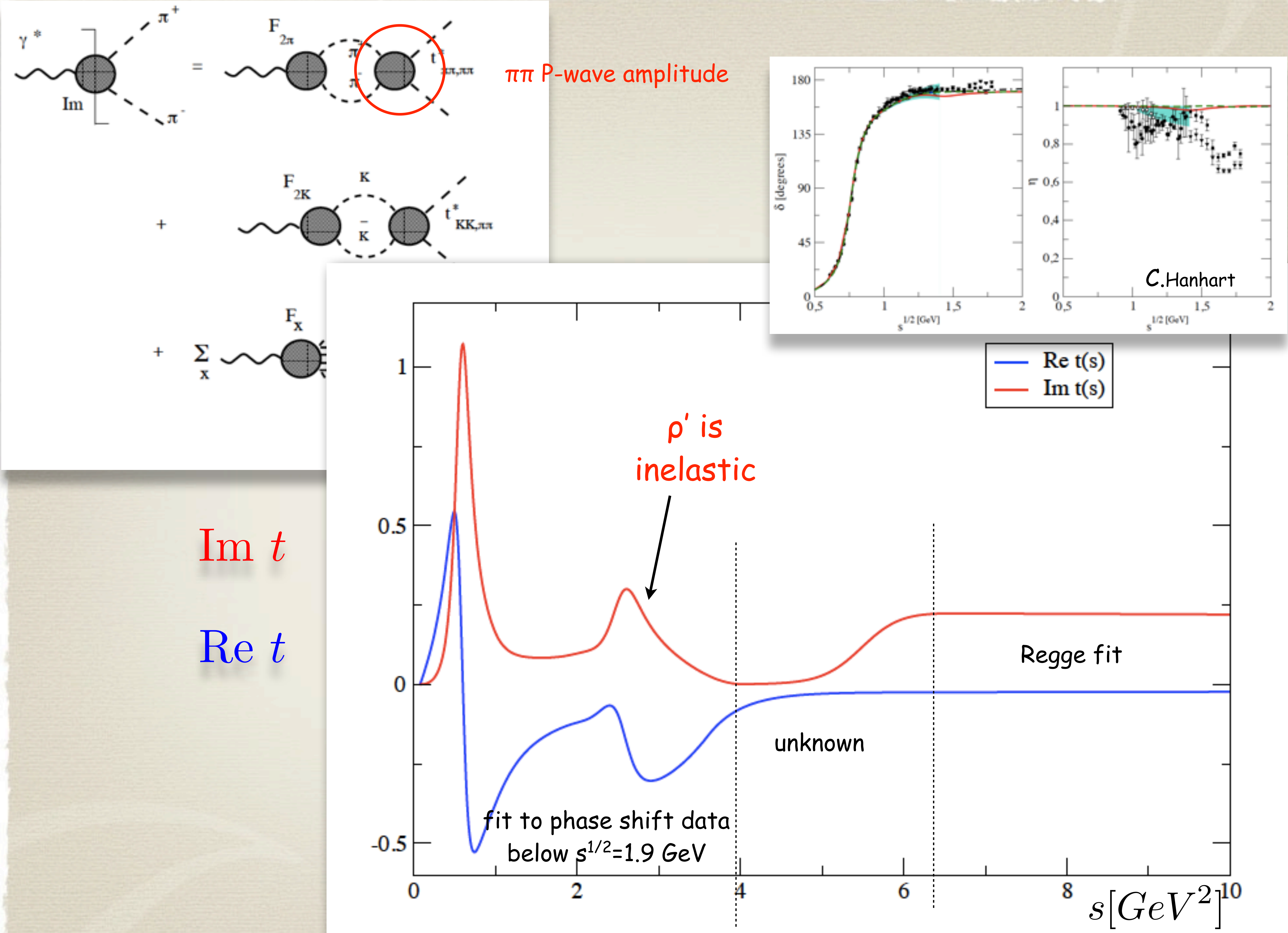
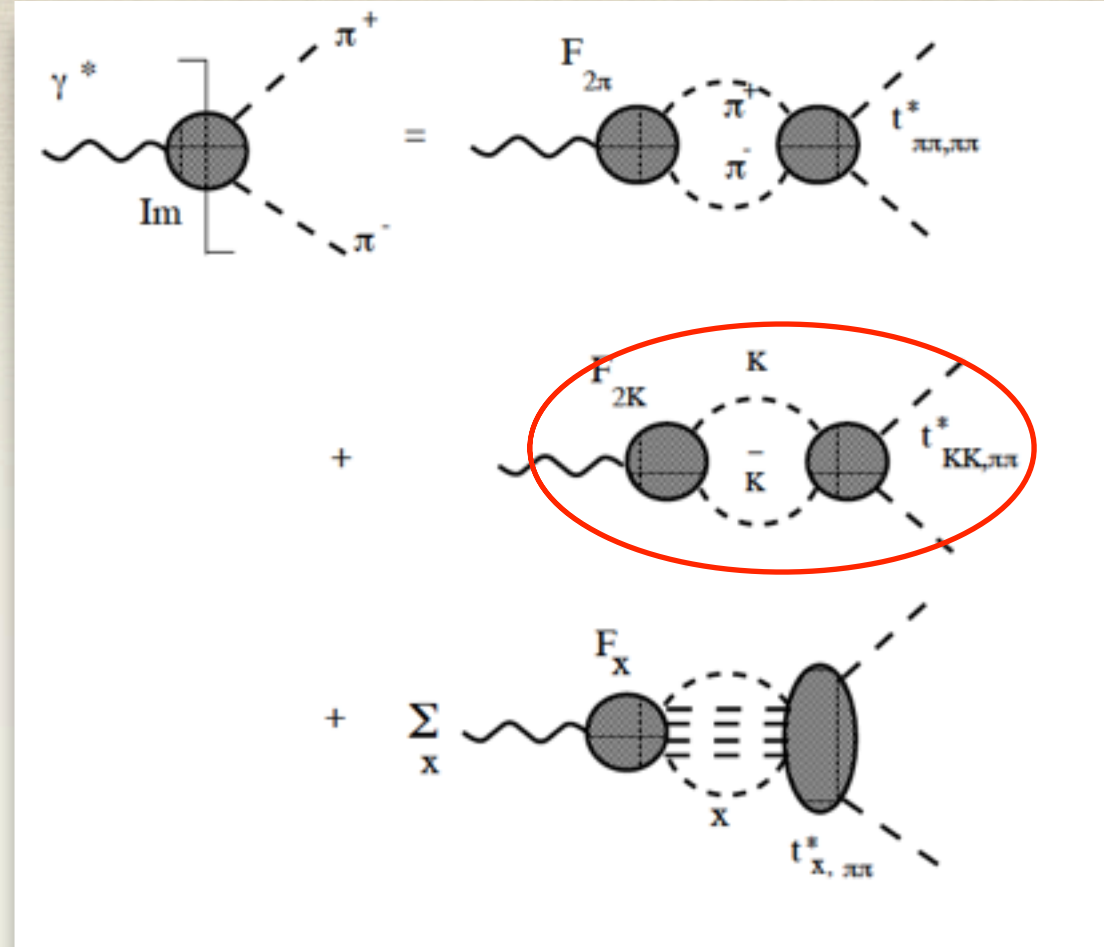



γ *
π +
π -
Im
=
F 2π
t ππ,ππ
ππ P-wave amplitude
F 2K
K
t* KK,ππ
+
+ Σ x
F x
Im t
Re t
Re t(s)
Im t(s)
ρ' is inelastic
Regge fit
unknown
fit to phase shift data below $s^{1/2}$=1.9 GeV
$s[GeV^2]$
C.Hanhart
δ [degrees]
η
$s^{1/2}$ [GeV]

## ✱ Inelastic contribution (I)

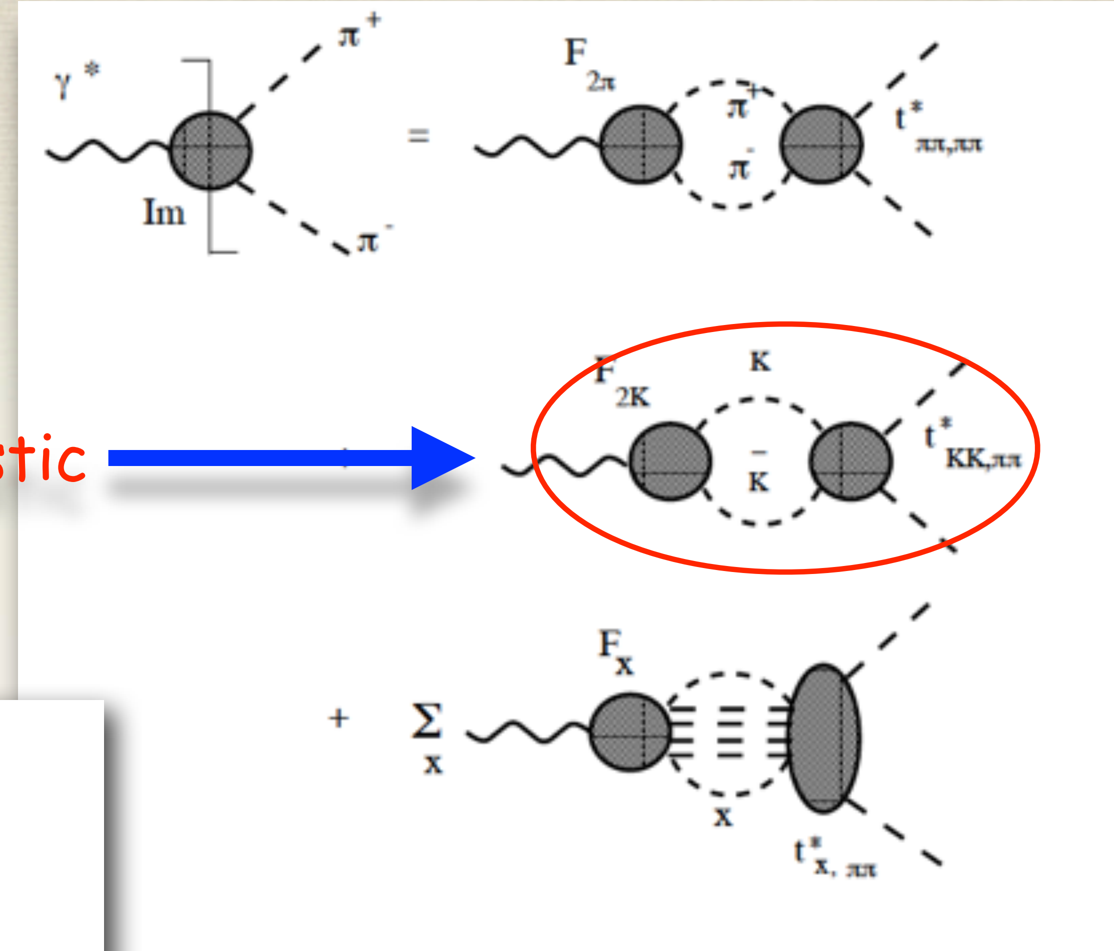
$$R = t^*_{2\pi,K\bar{K}}\rho_{2K}F_{2K} + \sum_X t^*_{2\pi,X}\rho_X F_X$$

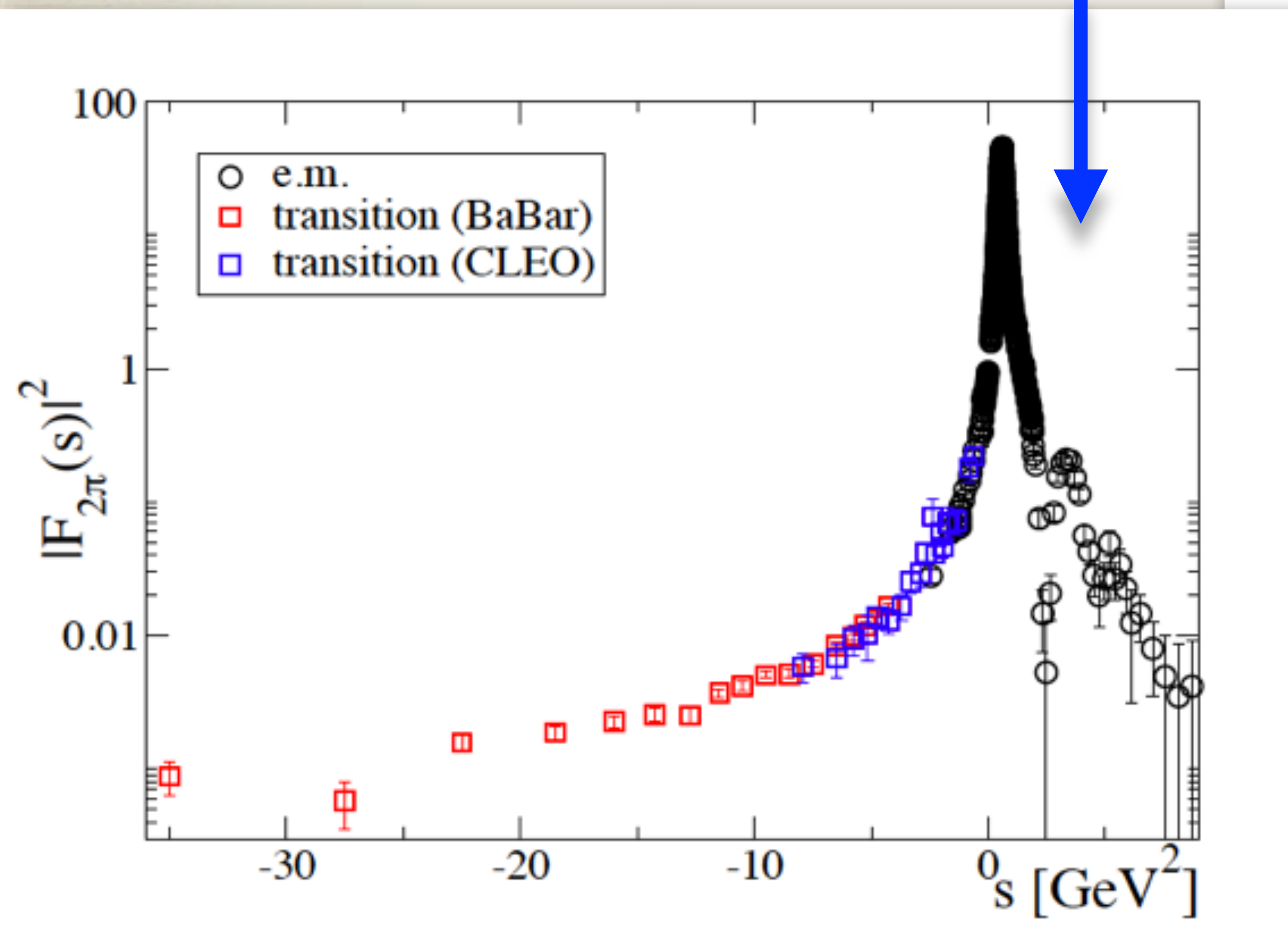


## * Inelastic contribution (I)

$$R = \underline{t^*_{2\pi,K\bar{K}}\rho_{2K}F_{2K}} + \sum_X t^*_{2\pi,X}\rho_X F_X$$

ρ' is inelastic



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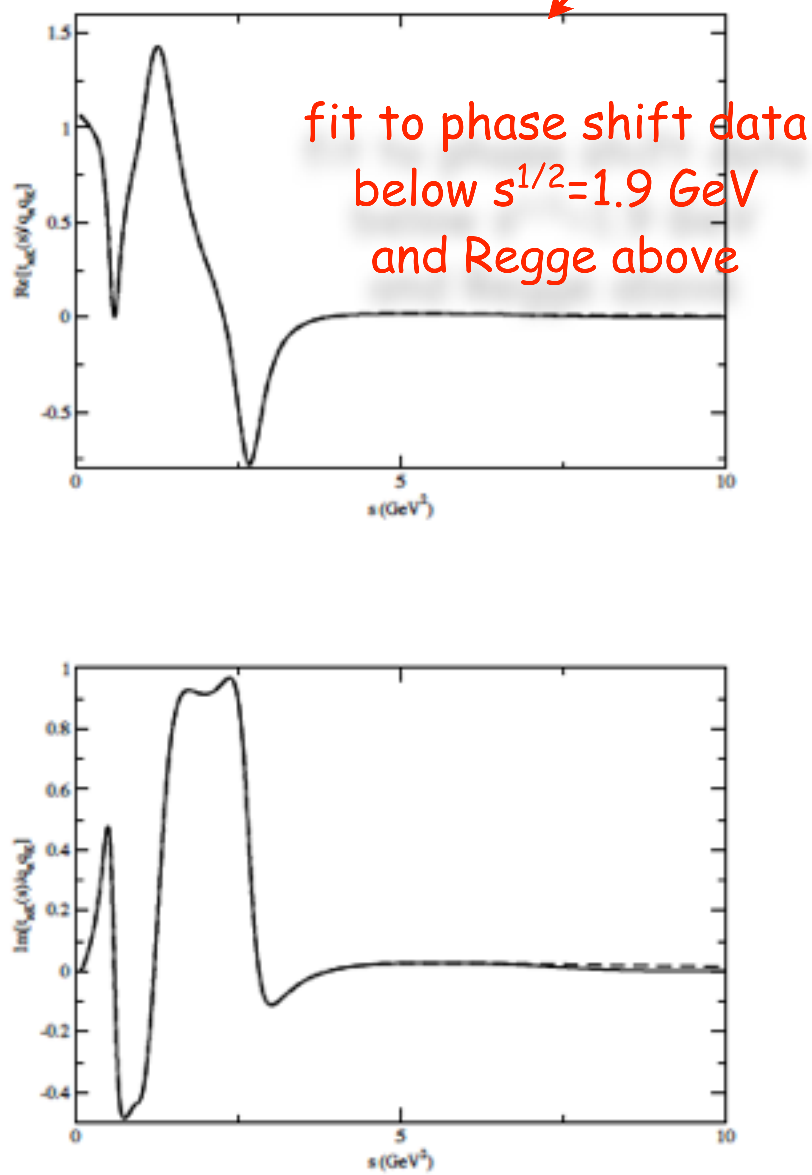
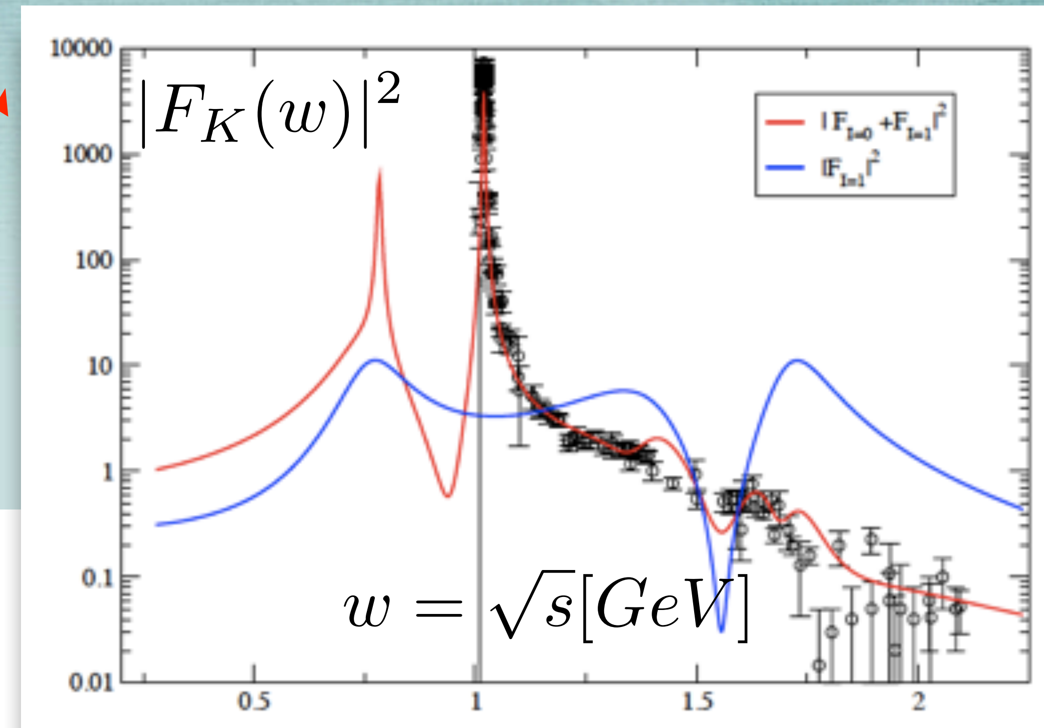
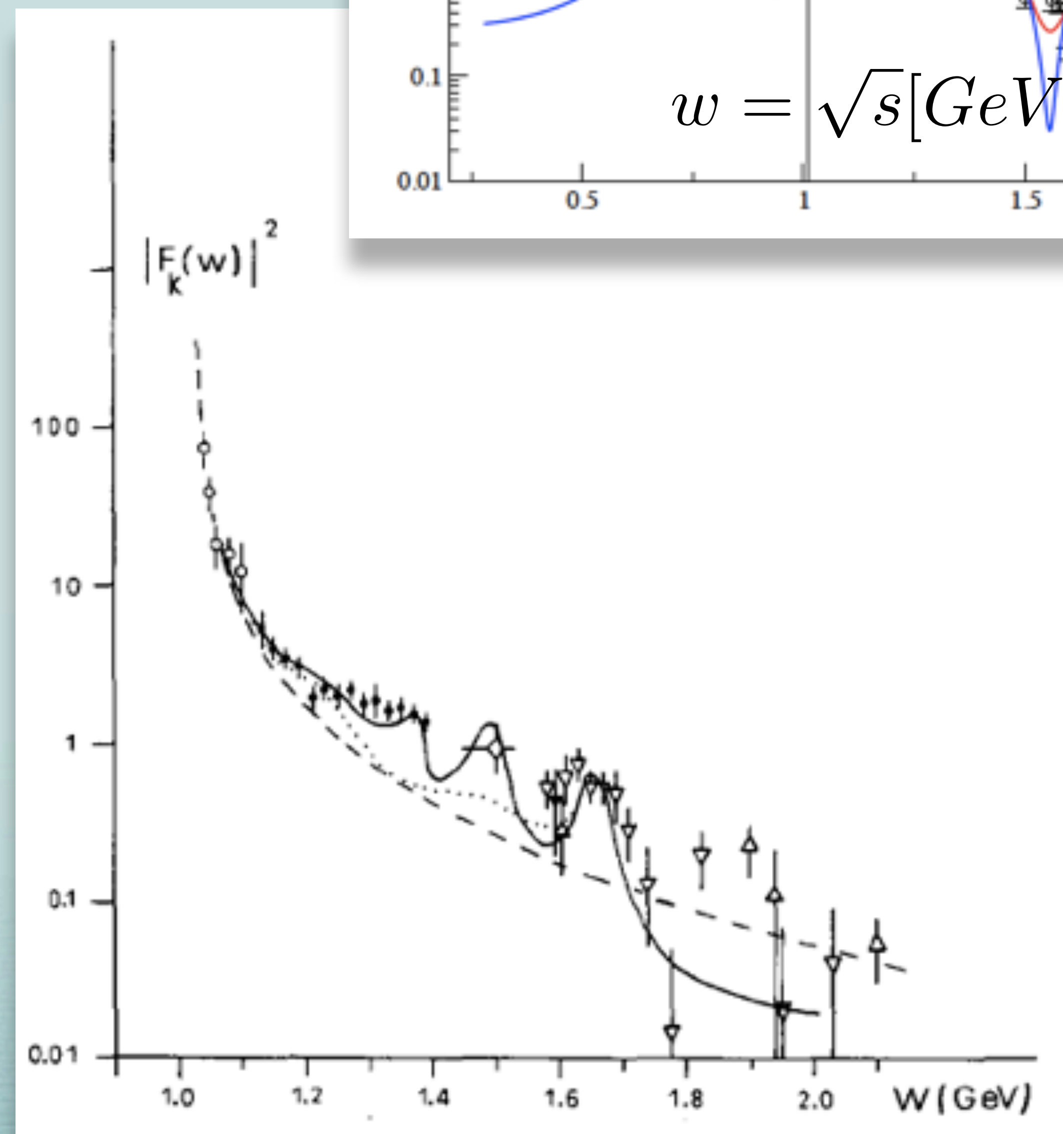


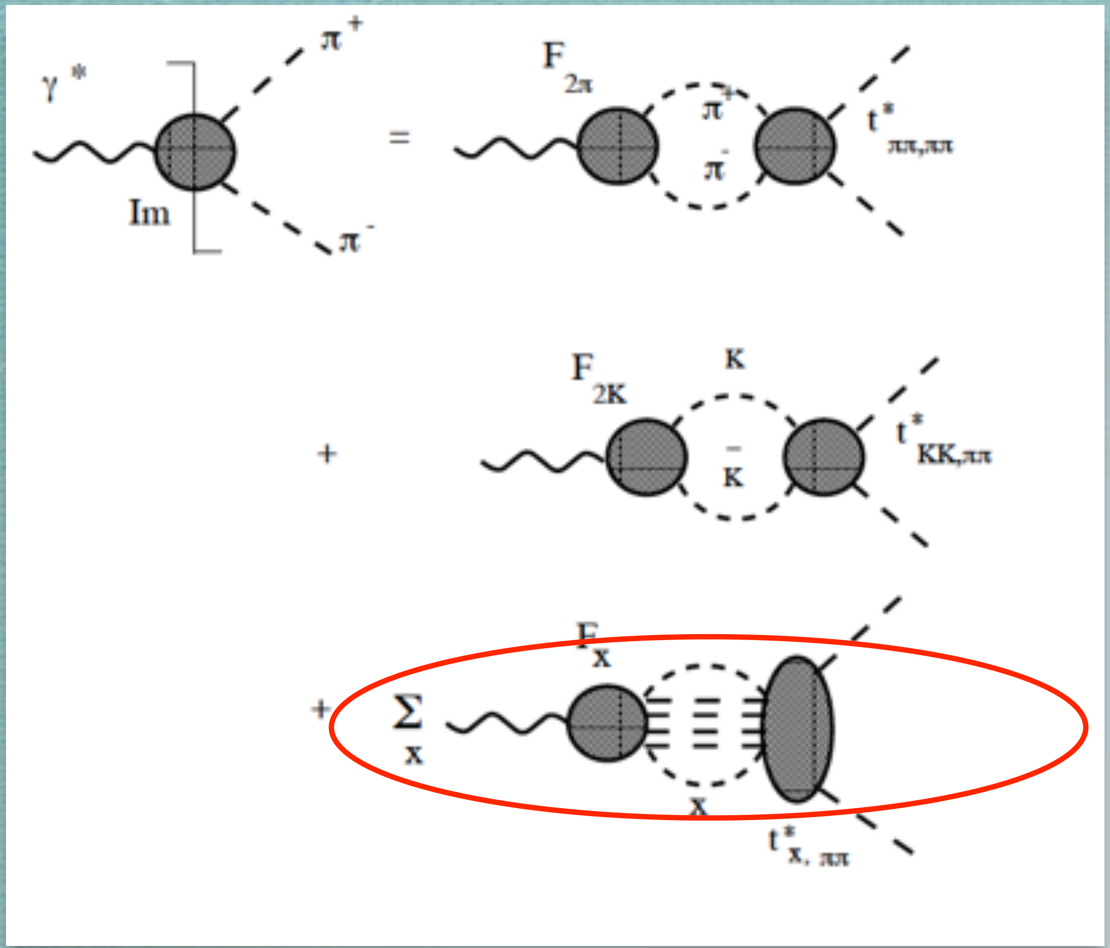

FIG. 8: Real (top) and imaginary (bottom) parts of the isovector, $P$-wave amplitude, $t_{\pi K}(s)/(q_\pi q_K)$ (solid lines). The dashed line is the result of the $K$-matrix parameterization.



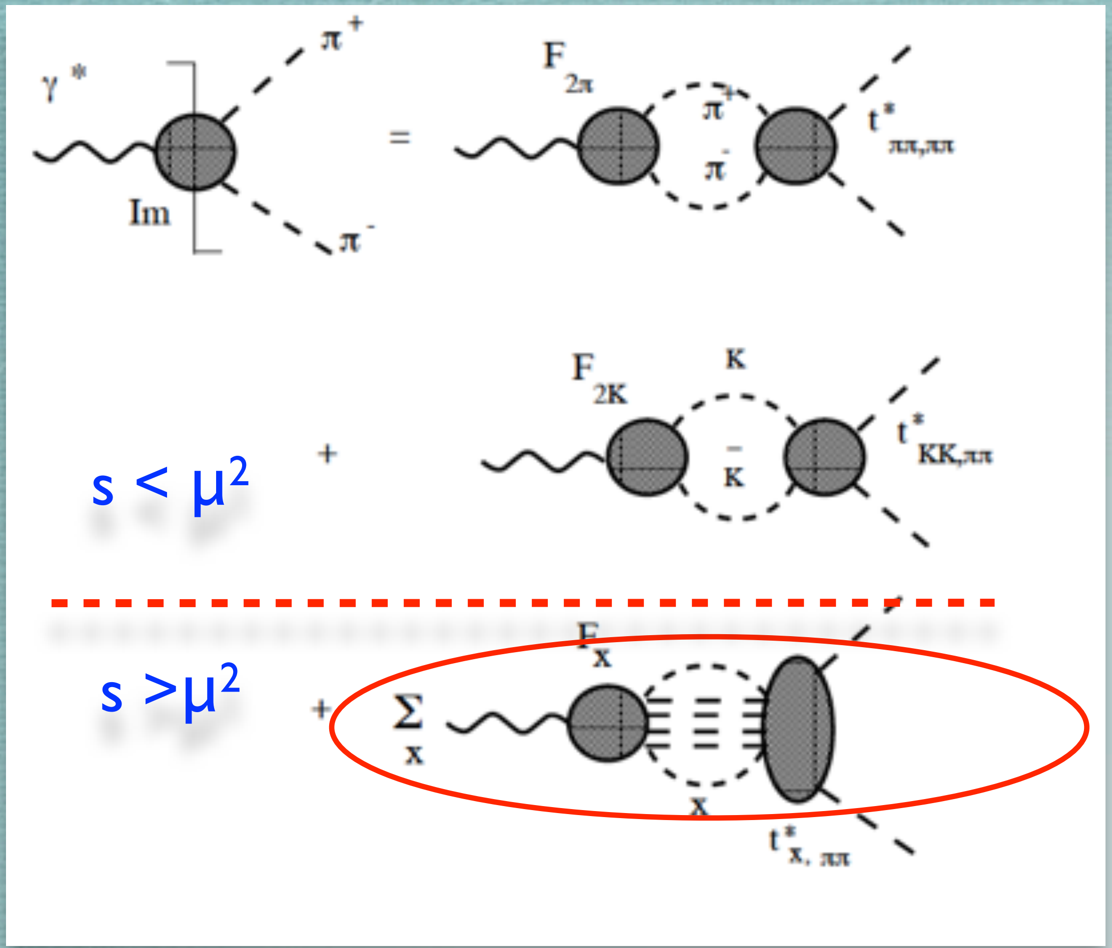
# Inelastic contribution (II)

$$R = t^*_{2\pi,K\bar{K}}\rho_{2K}F_{2K} + \sum_X t^*_{2\pi,X}\rho_X F_X$$

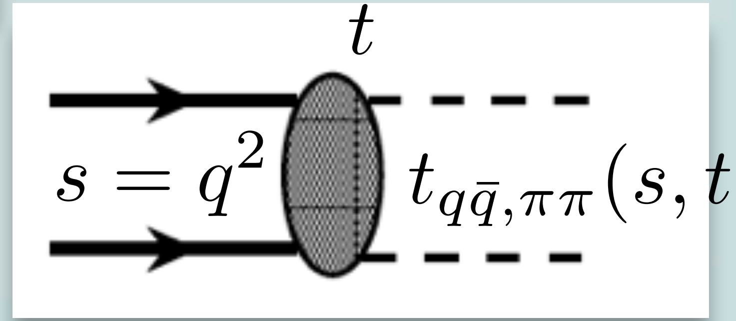


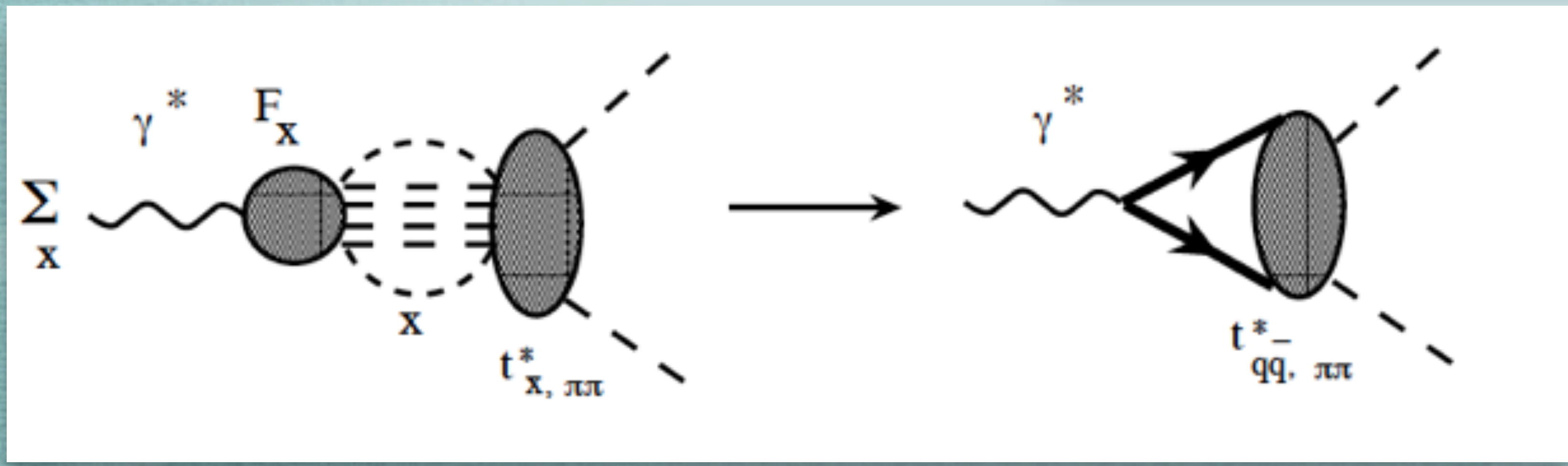
## * Inelastic contribution (II)

$$R = t^*_{2\pi,K\bar{K}}\rho_{2K}F_{2K} + \sum_X t^*_{2\pi,X}\rho_X F_X$$




$$\text{Im}\ t_{q\bar{q},\pi\pi} = \beta_\pi(t)s^{\alpha_q(t)}$$

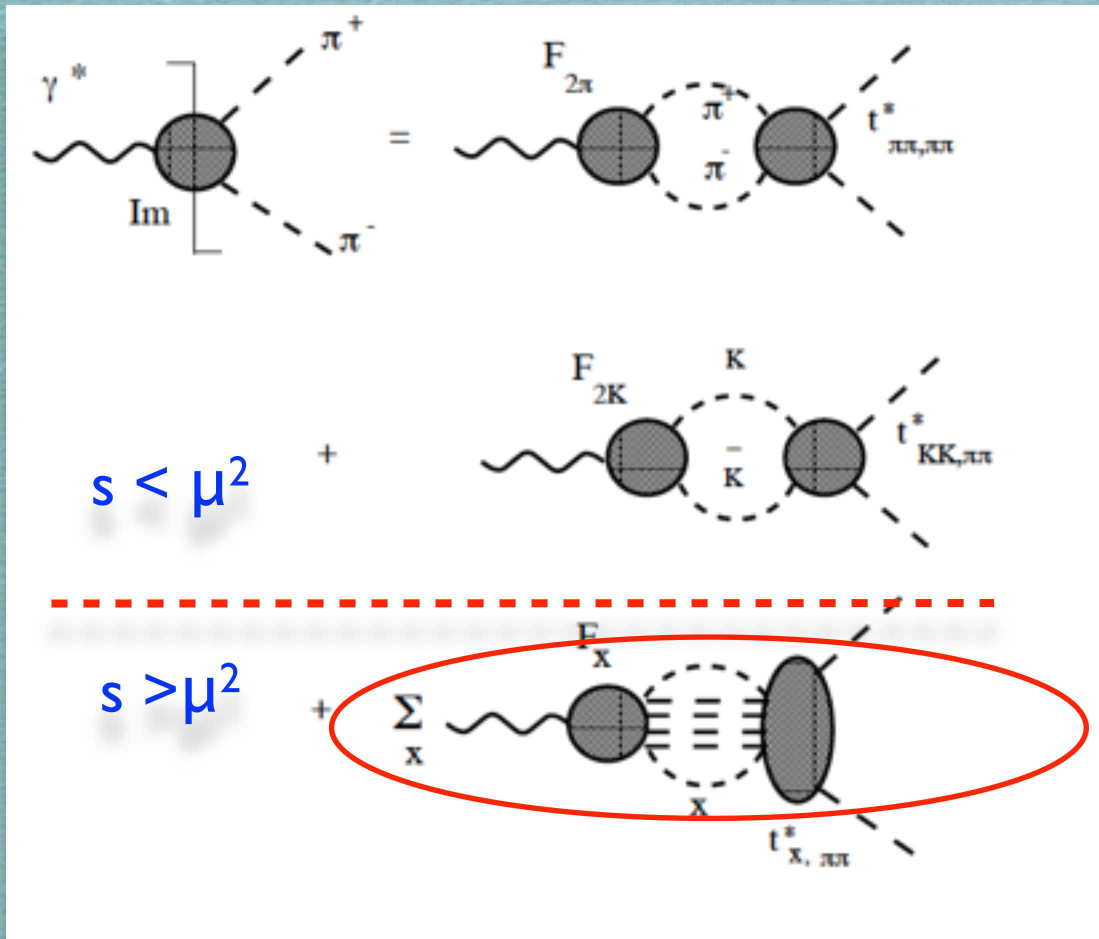



$$t_{q\bar{q},\pi\pi} = \int dz_t t_{q\bar{q},\pi\pi}(s,t)P_1(z_s)$$

# * Inelastic contribution (II)

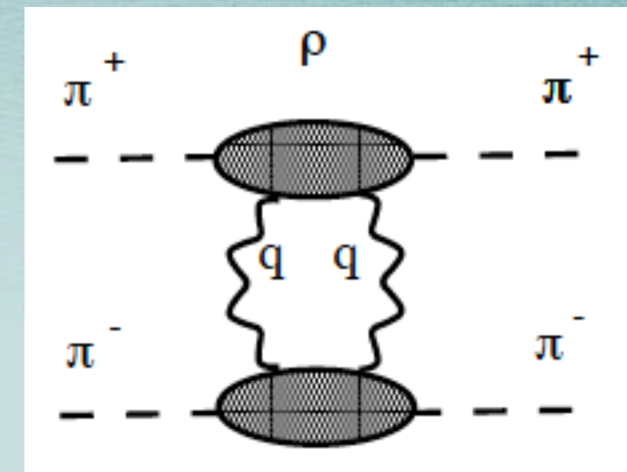
$$R = t^*_{2\pi,K\bar{K}}\rho_{2K}F_{2K} + \underline{\sum_X t^*_{2\pi,X}\rho_X F_X}$$




Mandelstam branchings

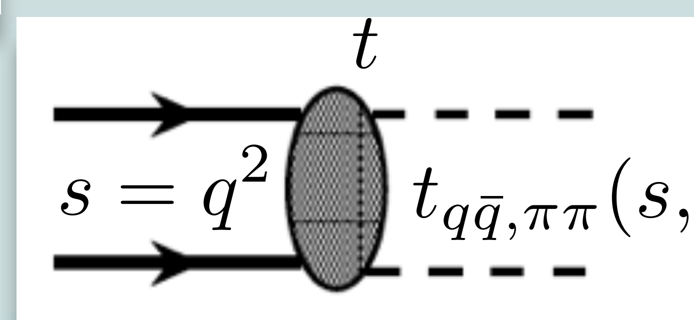
$$\alpha_q\left(\frac{t}{4}\right) \sim \frac{\alpha_\rho(t)+1}{2}$$

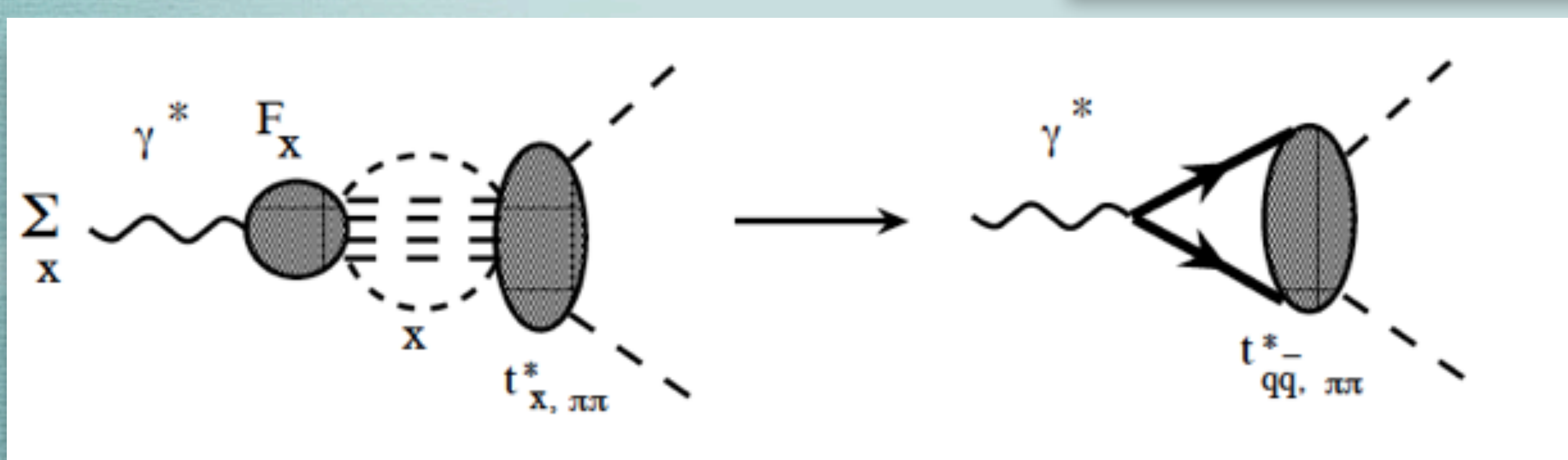
$$\alpha_q(t \sim 0) \sim 0.75$$




$$s\sum_{q\bar{q}} t^*_{2\pi,q\bar{q}}\rho_{q\bar{q}}F_{q\bar{q}} \sim s^{\alpha_q(0)-1/2}$$

$$\text{Im } t_{q\bar{q},\pi\pi} = \beta_\pi(t)s^{\alpha_q(t)}$$

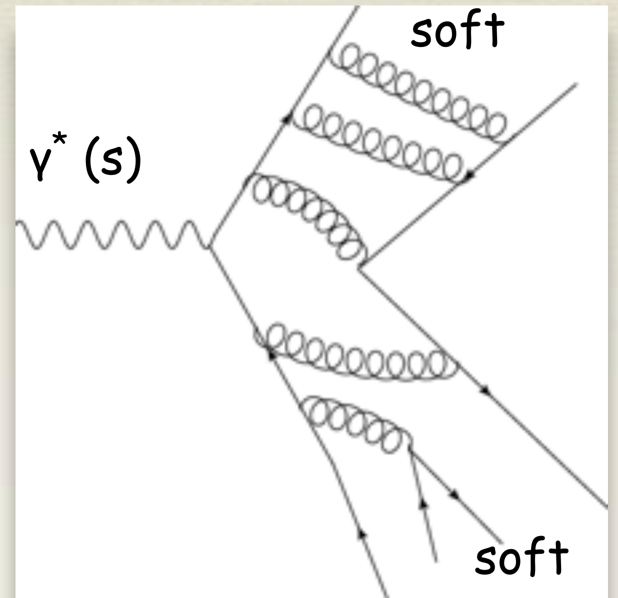


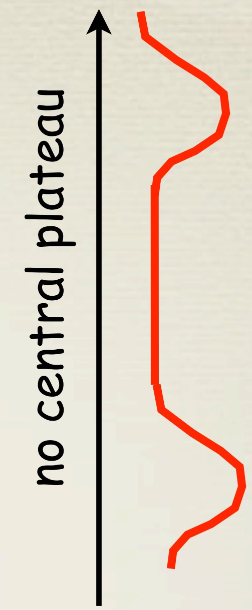




$$t_{q\bar{q},\pi\pi} = \int dz_t t_{q\bar{q},\pi\pi}(s,t)P_1(z_s)$$

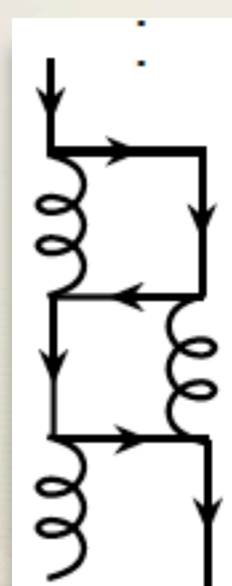
why reggezation enhances amplitudes

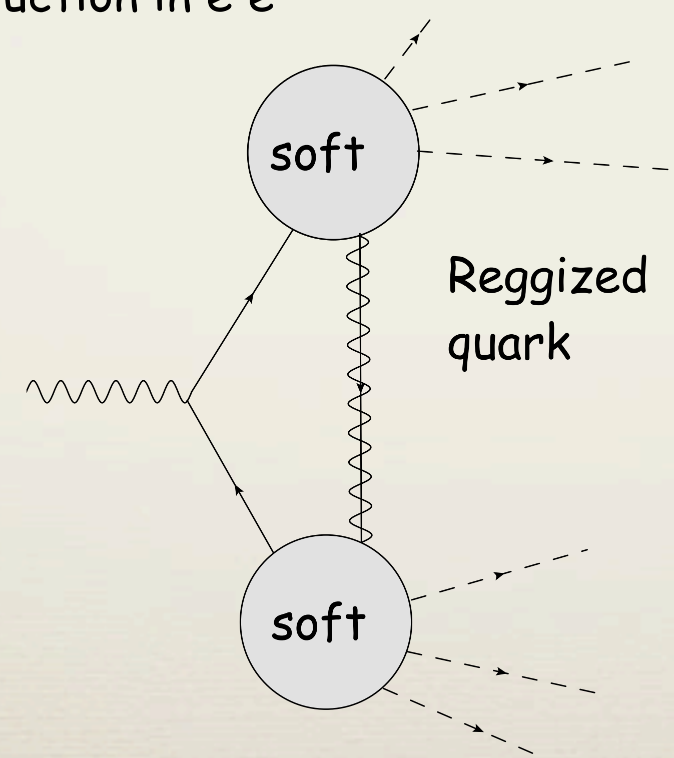
"leading Fock components"



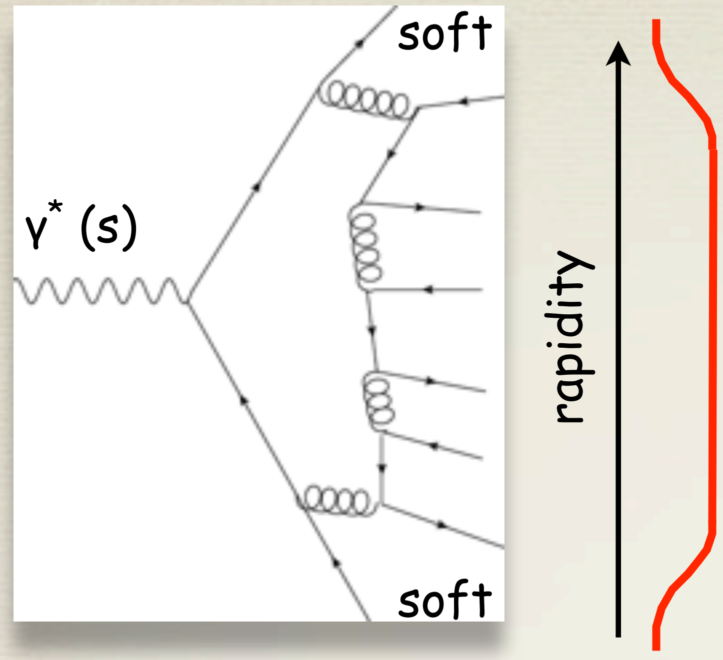

multi-particle production in $e^+e^-$

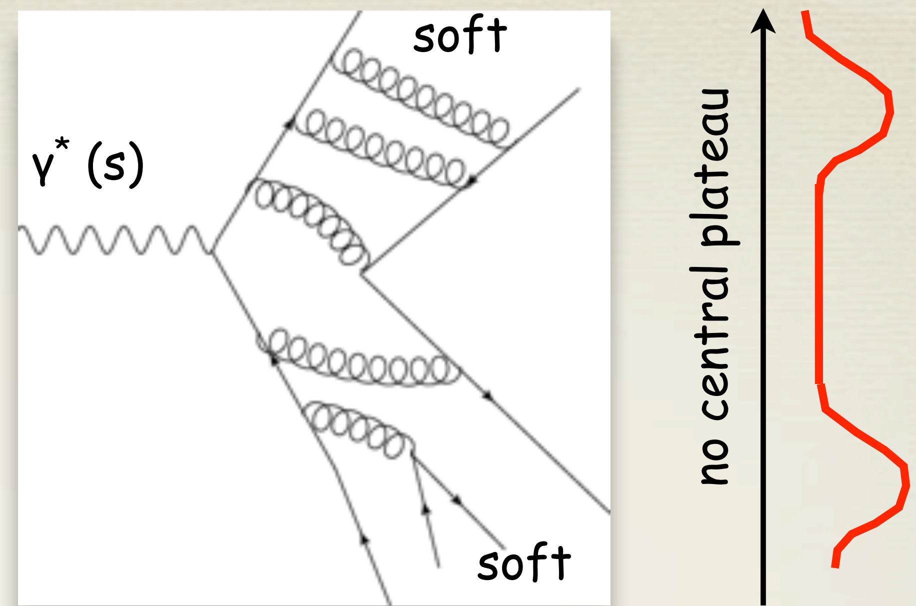




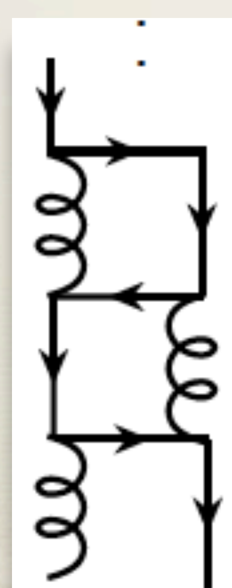
# why reggezation enhances amplitudes

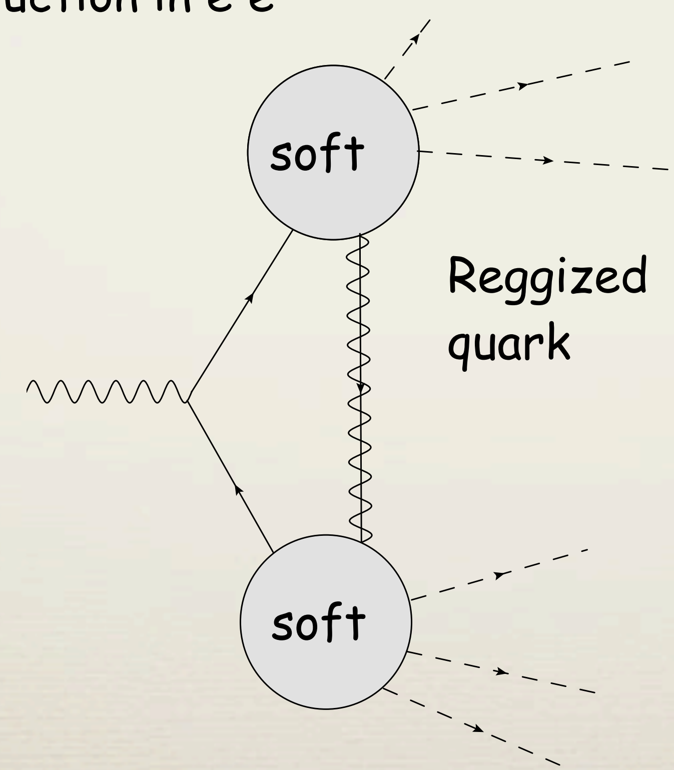
"leading Fock components"



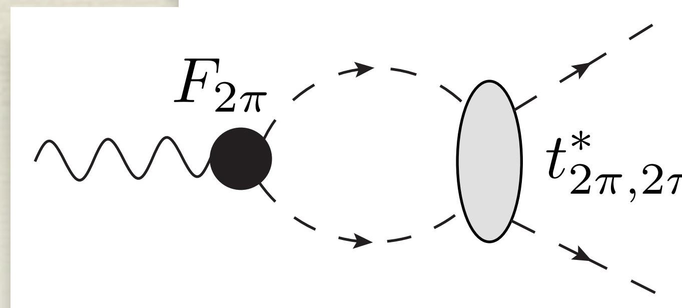

multi-particle production in $e^+e^-$



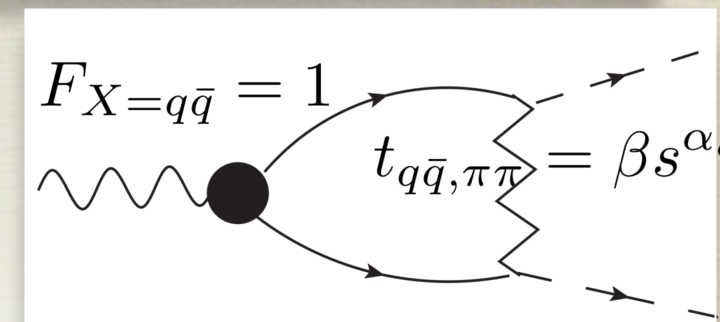


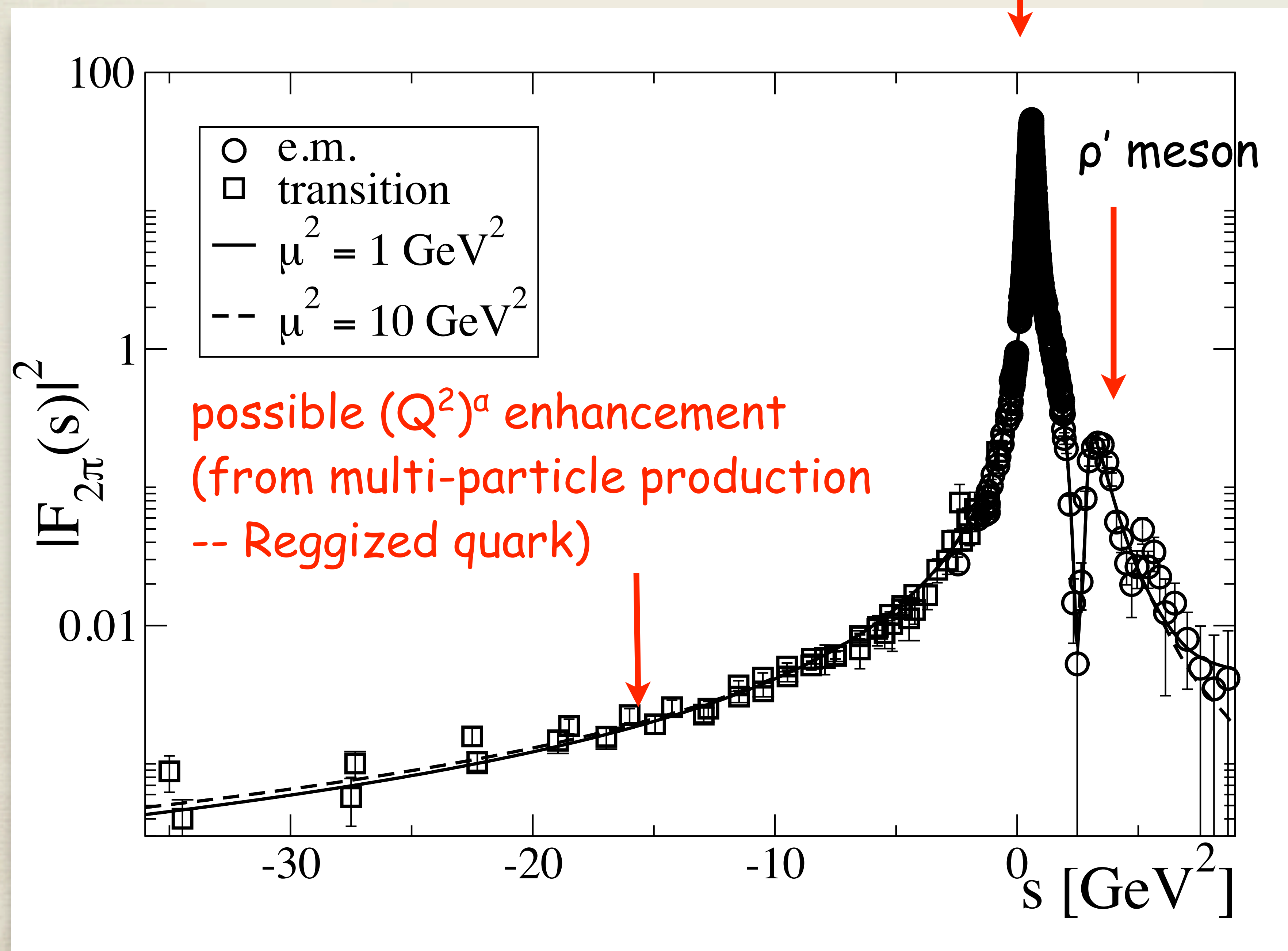
# pion e.m form factor (summary)

$$ImF_{2\pi} = t^*_{2\pi,2\pi}\rho_{2\pi}F_{2\pi} + t^*_{K\bar{K},2\pi}\rho_{2K}F_K + \sum_X t^*_{X,2\pi}\rho_X F_X$$




ρmeson



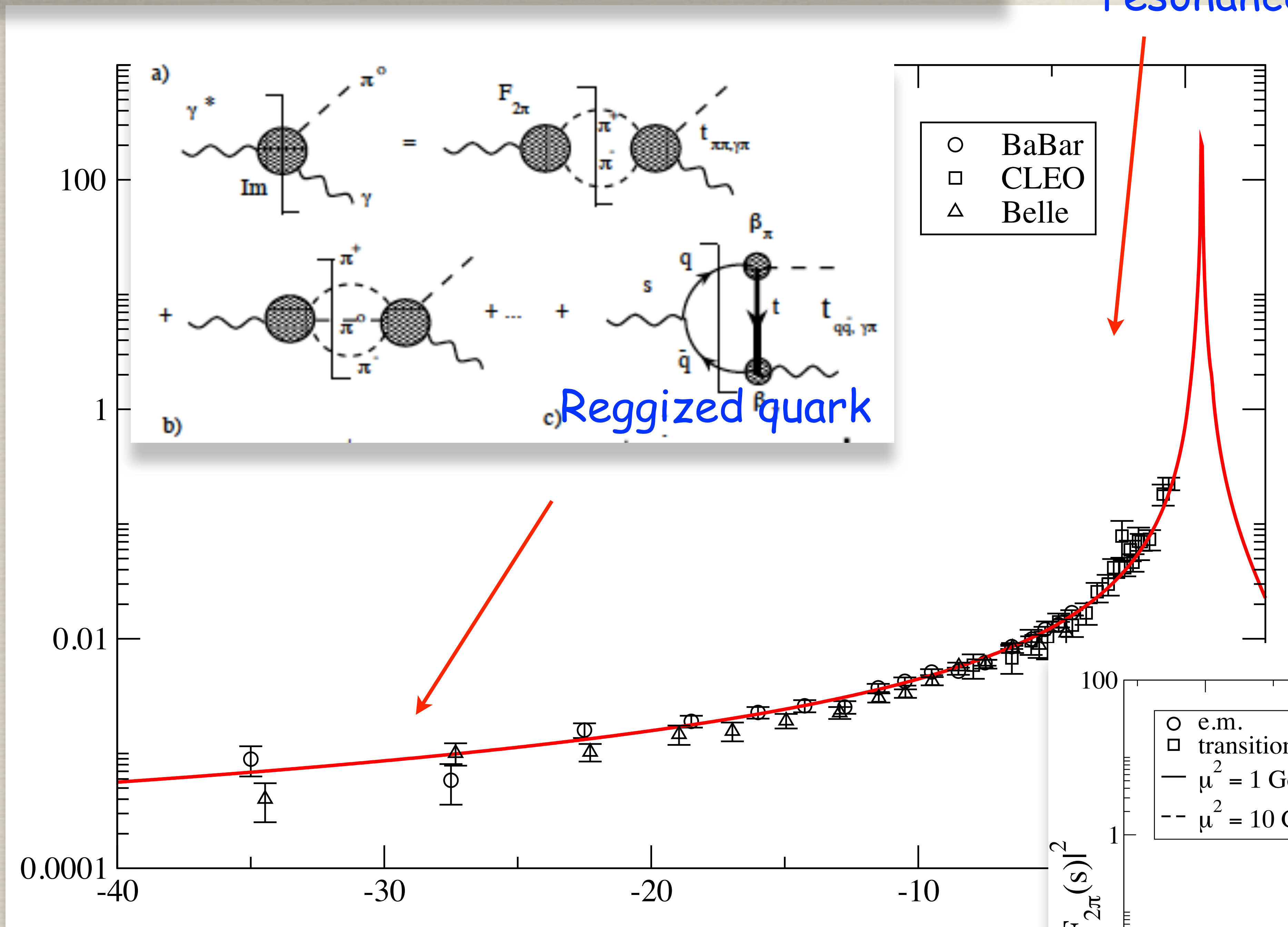



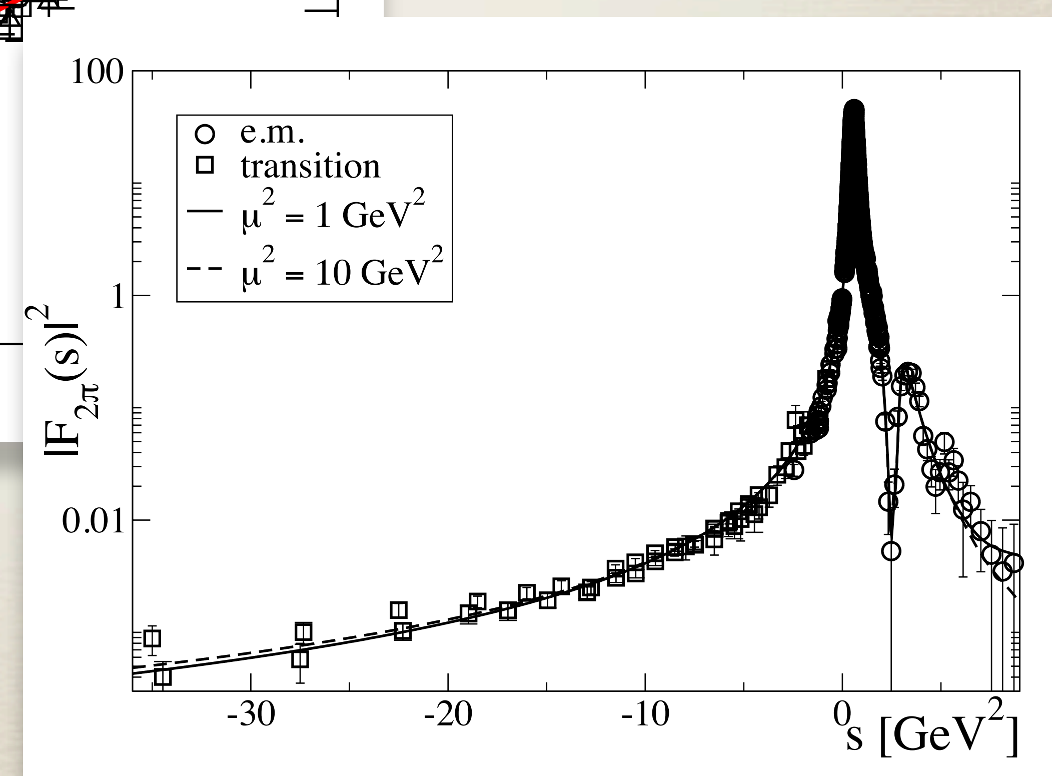

curves: dispersion relation solution with reggized quarks to describe large-s region

## pion transition form factor (summary)

$$ImF_{\pi\gamma} = t^*_{2\pi,\pi\gamma}\rho_{2\pi}F_{2\pi} + t^*_{3\pi,\pi\gamma}\rho_{3\pi}F_{3\pi} + \sum_X t^*_{X,\pi\gamma}\rho_X F_X$$

resonances (ω,ρ)



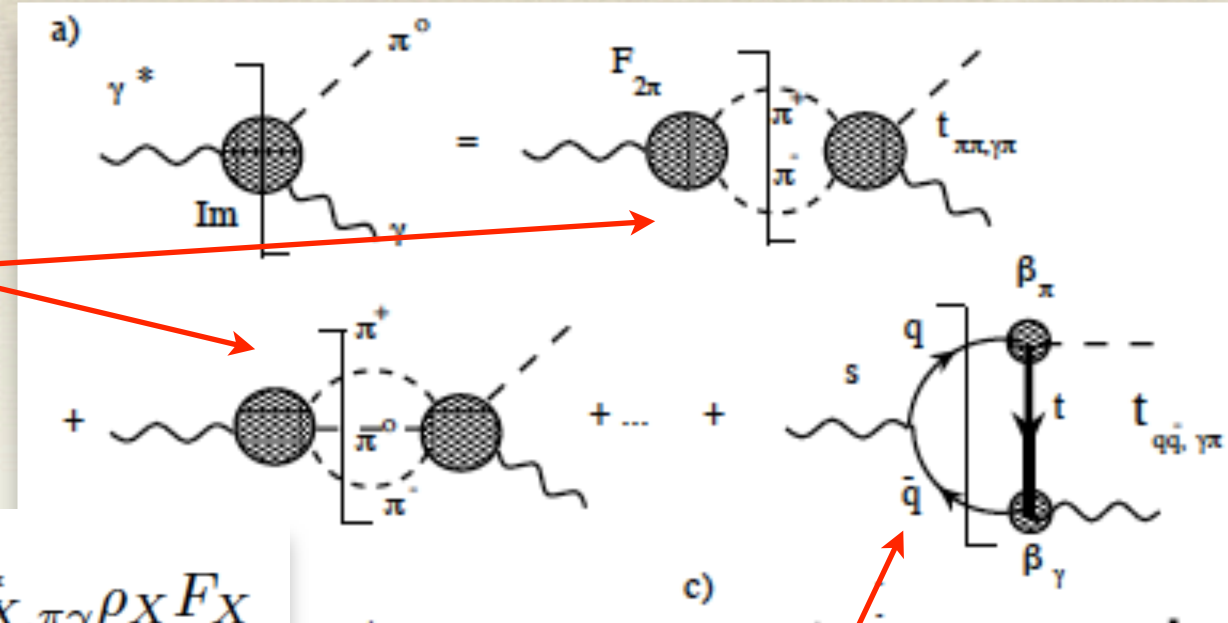



## From the s-channel:

resonances (ρ,ω) at low energies

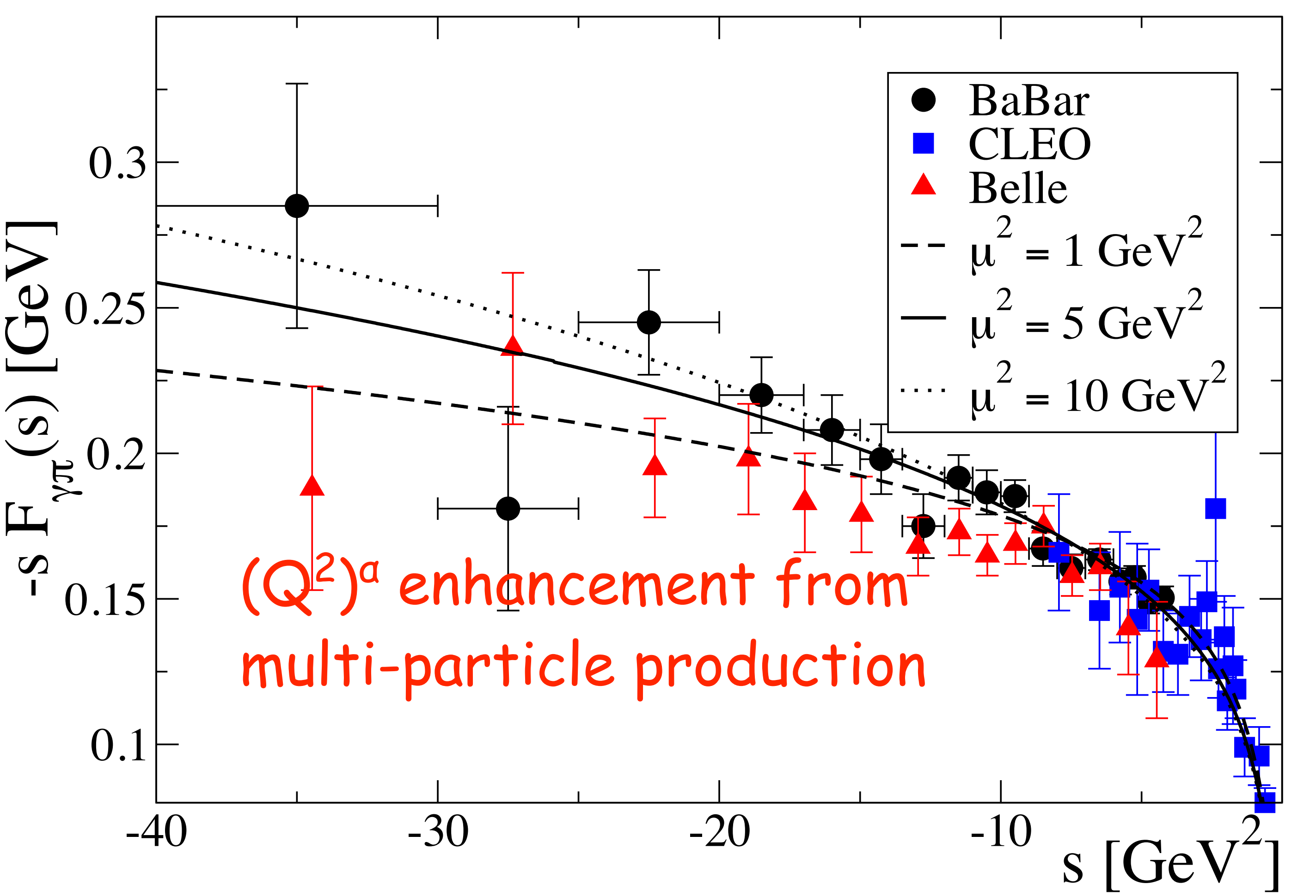
$$ImF(s) = \sum_X t^*_X(s)\rho_X(s)F_X(s)$$

$$ImF_{\pi\gamma} = t^*_{2\pi,\pi\gamma}\rho_{2\pi}F_{2\pi} + t^*_{3\pi,\pi\gamma}\rho_{3\pi}F_{3\pi} + \sum_X t^*_{X,\pi\gamma}\rho_X F_X$$




multi-particle ladder
-- Reggized quark (aka diffractive dissociation)

M.Gorchtein, P.Guo, A.P. Szczepaniak
arXiv:1102.5558 (PRC in press)




curves:
dispersion relation solution with reggized quarks to describe large-s region

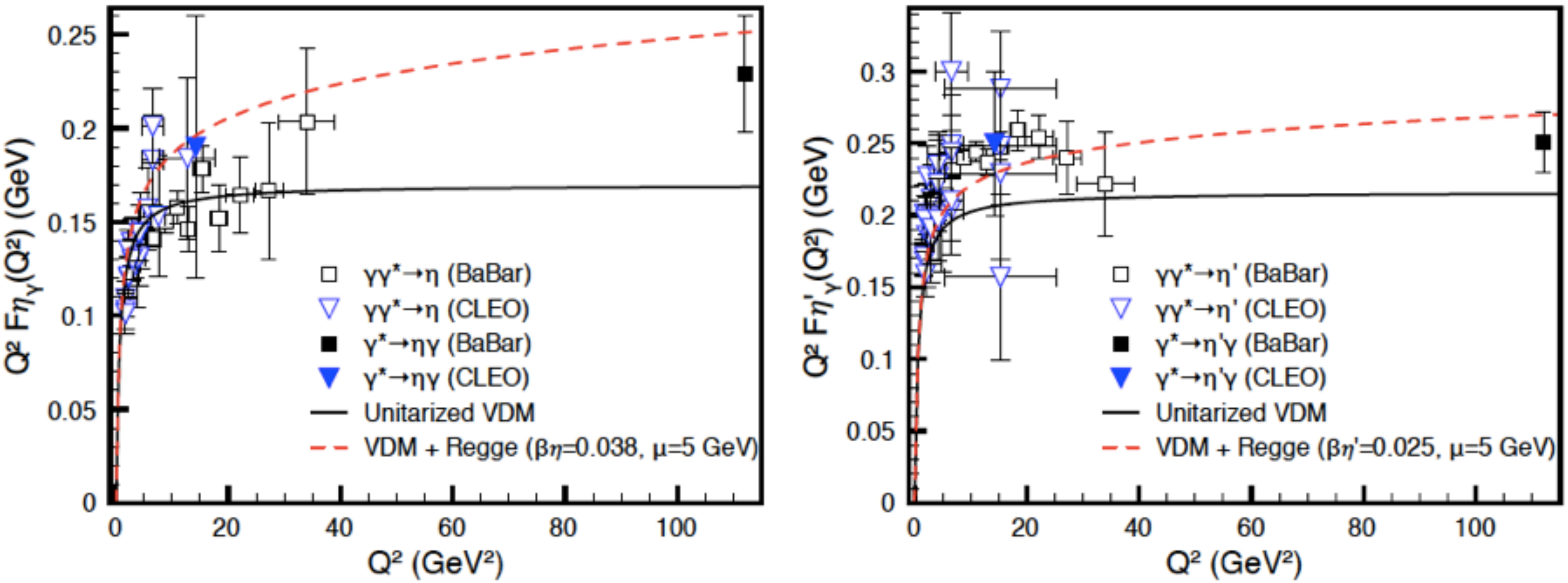



Figure 3: Left panel: experimental data for $|Q^2 F_{\eta\gamma}(Q^2)|$ in the space-like region from Refs. [23, 22] and high-$Q^2$ timelike data from Refs. [24] in comparison with unitarized VDM (solid line) and our full model (VDM + Regge) with $\mu^2 = 5$ GeV$^2$ (dashed line). Right panel: the same for $|Q^2 F_{\eta'\gamma}(Q^2)|$.

# Summary

* In the available energy range f.factors dominated by resonances
* Complete analysis requires self consistency: (e.g kaon form factor, Im part of inelasticity )
* Importance of Regge trajectories and not elementary particles